Easy exercises:

1. Show that any diagram of a knot with 0,1 or 2 crossings can be unknotted or it is trivial.
2. Find the linking coefficient of:
   a. The Hopf link;
   b. Borromean rings;
   c. The Whitehead link(find all three coefficients).
3. Find the number of tricolorings for:
   a. The right and the left trefoil knots;
   b. The figure-8 knot;

Exercises:

1. Show that the right and the left trefoil-knots are both invertible.
2. Show that the figure-eight knot is both invertible and amphicheiral(equivalent to its mirror image)
3. Show that you can turn any diagram of any knot into an unknot by switching some crossing types (from overcrossings to undercrossings and vice verca)
4. Let the unknotting number be M. Prove that $M \geq log_3(c_3(K)) - 1)$, where c_3 is the number of Fox 3-colourings.

   Show that the unknotting number for the trefoil knot and figure-eight knot are equal to 1 ;

   Show that the unknotting number of the 7_4 knot does not exceed 2.
5. Show that the number of p-colorings of any link equals p^n for some natural number n.
6. Let $L = L_1 \sqcup L_2$ be an oriented linking. Show that $lk(L_1, L_2) = lk(L_2, L_1)$
7. Let $L = L_1 \sqcup L_2$ be an oriented linking. Show that $-\ lk(L_1, L_2) = lk(-\ L_1, L_2)$
8. Show that all polygonal links with less than six edges are trivial.
9. Draw a polygonal trefoil knot with six edges.
10. Show that if a link is splittable, then it has the linking number 0. Is it true that any link with linking number 0 is splittable? If yes, explain why and if not, provide a counterexample.

Hard exercises:

1. Show that all three Reidemeister moves are independent, meaning that one can not derive any one from the other two.
2. Find a trivial knot diagram that doesn't admit any decreasing Ω_1 and Ω_2 and neither admits any Ω_3.

   In other words, find a non-trivial (locally minimal) diagram of the unknot.

   (Article: "Every Reidemeister move is needed for each knot type" by J.Hagge, 2004)

Problems:

1. Given two diagrams K and K' of the same knot having n and m crossings, respectively. Estimate the number of Reidemeister moves needed to pass from K to K'
2. Estimate the crossing number in the intermediate knot diagram that appears between the equivalent knot diagrams with n and m intersections.