Gravitational Collapse and Spacetime Singularities

1 Goal:

The series of seminars aims to achieve two primary objectives: first, to develop a rigorous mathematical understanding of why the formation of singularities becomes generically inevitable under certain conditions; and second, to explore existing approaches for investigating the causal nature of singularities that arise during a gravitational collapse.

2 Prerequisite:

Graduate-level knowledge in General Relativity and differential geometry.

3 List of topics to be discussed:

- **Introduction**: Kinematics of a deformable medium, the decompositions of deformation tensor, expansion, shear, and rotation.

- **Raychaudhuri's equation and Focusing theorem:** Propagation of deviation vectors along a timelike curve, the evolution of expansion scalar, and the components of shear and rotation tensor along the timelike curve, Frobenius' theorem, Raychaudhuri's equation, and focusing theorem, caustic point of timelike geodesic congruence, energy condition, Raychaudhuri's equation, and focusing theorem for null geodesic congruence, the physical meaning of expansion scalar, behavior of expansion scalar in some exact solutions of Einstein's equations.

- **Conjugate points**: Definition of conjugate points, variation of arc length of timelike geodesic, some important theorems regarding conjugate points and proofs of them.

- **Global Causality condition of spacetime manifold** Orientability, causal curves, the notion of causal future/past and chronological future/past, achronal boundaries, global causality conditions, avoidance of closed timelike curves, strong causality condition, stable causality condition, domain of dependence, Cauchy surface, Cauchy horizon, imposition of compactness condition on the domain of dependence, global hyperbolicity, some important theorems and their proofs regarding the global hyperbolicity.

- **Singularity theorems** The definition of singularities, geodesic incompleteness, singularity theorem I (with non-compact Cauchy surface) and its proof, singularity theorem II (without non-compact Cauchy surface) and its proof, nature of singularities, cosmic censorship conjecture.

- **Gravitational Collapse**: Homogeneous collapse, spacelike singularities, inhomogeneous collapse, formation of Cauchy horizon, and non-spacelike singularities in inhomogeneous gravitational collapse, Christodoulou's result (Annals of Mathematics, 149 (1999), 183-217) on the instability of naked singularities in the gravitational collapse of scalar fields.

4 Keywords:

Deformation tensor, Raychaudhuri's equation, focusing theorem, energy condition, null geodesic congruence, conjugate points, causal curves, chronological future/past, achronal boundaries, global hyperbolicity, domain of dependence, Cauchy surface, Cauchy horizon, trapped surfaces, singularity theorems, future trapped and past trapped achronal set, cosmic censorship conjecture, gravitational collapse, naked singularities, local and global visibility of singularities, scalar field collapse.

5 References:

1. General Relativity, Robert M. Wald, The University of Chicago Press (1984).

2. The Large Scale Structure of Space-Time, S.W. Hawking, G.F.R. Ellis, Cambridge Monographs on mathematical physics, Cambridge University Press (2023 new edition).

3. Gravitational collapse and spacetime singularities, Pankaj S. Joshi, Cambridge monographs on mathematical physics, Cambridge University Press (2007).

4. A Relativist's toolkit, The mathematics of black hole mechanics, Eric Poisson, Cambridge University Press (2004).

5. Spinor and Spacetime, Volume I, R. Penrose, W. Rindler, Cambridge University Press (1984).

6. The Mathematical Theory of Black Holes, S. Chandrasekhar, The International Series of Monographs on Physics.

7. Instability of naked singularities in the gravitational collapse of scalar fields, Christodoulou, Annals of Mathematics, **149** (1999), 183-217.

8. Stability and instability of the Cauchy horizon for the spherically symmetric Einstein-Maxwell-scalar field equations, Mihalis Dafermos, Annals of Mathematics, **158** (2003), 875–928.

6 Audience:

Graduate, Postdoc, Researcher.

7 Mode of the discussion, venue and time:

Offline, in English, at the Shuangqing Building, at Tsinghua University.
During the Spring semester 2025. Every Wednesday (starting February 19, 2025) from 16:30 to 18:30.

8 Organizers:

Dipanjan Dey (Postdoc, BIMSA),
Sari Ghanem (Visiting Assistant Professor, BIMSA).