FLATTENING THEOREM FOLLOWING RAYNAUD-GRUSON AND GUIGNARD

AHMED ABBES

The goal of the lectures is to present Quentin Guignard's new proof of the following theorem of Raynaud-Gruson:

Theorem 1 ([4] I.5.2.2, [2] 1.1)**.** *Let X be a coherent scheme (i.e., quasi-compact and quasi-separated), $f\colon Y \to X$ a morphism of finite presentation, U a quasi-compact open subset of X and $\mathscr{F}$ a quasi-coherent $\mathscr{O}_Y$-module of finite type. Assume that the restriction of $\mathscr{F}$ to $Y \times_X U$ is a finitely presented $\mathscr{O}_{Y\times_X U}$-module which is flat over U. Then, there exists a blow-up $\varphi\colon X' \to X$ such that:*

(i) *The center of the blow-up φ is a finitely presented closed subscheme of X, disjoint from U.*
(ii) *If Y' is the strict transform of the X-scheme Y along φ, then the strict transform $\mathscr{F}'$ of $\mathscr{F}$ along φ is finitely presented over $\mathscr{O}_{Y'}$ and flat over X'.*

This theorem is intimately related to rigid geometry [1, 3]. A variant for formal schemes plays a crucial role for the study of flatness in rigid geometry, which was Raynaud's main motivation. Moreover, Guignard's new proof unravels important features of rigid varieties, particularly their local structure, shedding new lights on the theory of adic spaces.

These lectures are for students who have a basic knowledge in algebraic geometry. No knowledge of rigid geometry is required as I will not discuss the formal/rigid variant of the theorem.

References

[1] A. Abbes, Éléments de géométrie rigide. Volume I. Construction et étude géométrique des espaces rigides, Progress in Mathematics Vol. **286**, Birkhäuser (2010).

[2] Quentin Guignard, *A new proof of Raynaud-Gruson's flattening theorem*, International Mathematics Research Notices rnz042 (2019) (arXiv:1801.01188).

[3] M. Raynaud, *Géométrie analytique rigide d'après Tate, Kiehl,... Table ronde d'analyse non archimidienne*, Bull. Soc. Math. Fr. Mém. **39/40** (1974), 319-327.

[4] M. Raynaud, L. Gruson, *Critères de platitude et de projectivité. Première partie : Platitude en géométrie algébrique*, Invent. Math. **13** (1971), 1-89.