2022年清华大学数学中心暑期学校

1. Title: Dimer models in statistical mechanics: exact solution, combinatorics, Markov sampling（6次课，12学时）

主讲人 Speaker: Nicoli Reshetikhin & Emily Bain(YMSC)

2. Title: **代数拓扑**（9次课，18学时）

主讲人 Speaker: 吴杰(BIMSA)

3. Title: Introduction to geometric representation theory（9次课，18学时）

主讲人 Speaker: 李鹏辉(YMSC)

4. Title: **冯诺依曼代数**（9次课，18学时）

主讲人 Speaker: 吴劲松 (BIMSA)

5. Title: Introduction to p-adic linear differential equations（9次课，18学时）

主讲人 Speaker: Tinhinane Amina Azzouz(BIMSA)

6. Title: **多粒子模型简介**（6次课，12学时）

主讲人 Speaker: 蔚辉(YMSC)

7. Title: **速度和精度的学问——计算数学纵览**（8次课， 16学时）

Speed and Accuracy - An Introduction to Computational Mathematics

主讲人 Speaker: 王珺(YMSC)

课程介绍

1. Title: Dimer Models

主讲人 Speaker: Nicoli Reshetikhin & Emily Bain(YMSC)

This is a short introduction to dimer models in statistical mechanics
and the combinatorics that is involved is their solution.

Course description: Dimer models appeared in statistical mechanics in the early 1960's. These are models in equilibrium statistical mechanics on a two dimensional lattice where the interaction is determined by certain combinatorial exclusion rules. As in any model of equilibrium statistical mechanics the main challenge is to find the characteristics of the model, including its correlation functions in the limit when the "size of the system" goes to infinity.

Tentative plan of the mini-course is as follows:

Lecture 1. (NR) Short introduction to equilibrium statistical mechanics of lattice models. Dimer models on a graph. Correlation functions. Examples. The thermodynamics limit.

Lecture 2. (NR) Dimer models in statistical mechanics. Equivalent combinatorial objects: tilings, lattice path. Kasteleyn solution to a dimer model on a plane graph.

Lecture 3. (NR) Dimer models on large domains. The thermodynamic limit for dimer models on lattices.

Lecture 4. (NR) The limit shape phenomenon in dimer models on bipartite lattices.

Lecture 5. (EB) How to study dimer models numerically? Introduction to Markov processes (special type of stochastic processes).

Lecture 6. (EB) Sampling dimer models using Markov processes.

2. Title: 代数拓扑

主讲人 Speaker: 吴杰(BIMSA)

Course description: The course of algebraic topology will be designed in the three levels (beginning level, graduate level and research level) with emphasizing on interactions between students, lecturer and teaching assistants, carried out by lectures, student reports, and junior researcher talks, on some selected topics supplement to the usual textbooks of algebraic topology including but not restricted to the following upon student's background and demands:

Volume of simplex. Weak topology, metrization theorem and paracompact theorem of (infinite) simplicial complexes. Delta set and simplicial set. Nerve complex and path complex. Reviews on simplicial homology.

Topological data analysis. Homotopy Theory Simplicial group theory.

Homotopy aspects of knot theory.

In addition, there will be selected topics as projects explored by the students in study groups including but not restricted to the following upon student's interests:

Quantifiable functions and size functions on simplicial complexes.

Probability aspects of simplices and simplicial complex.

Complex network, structures of graphs, clique complexes, neighborhood complex and path complex. Weighted simplicial complex. Topological approaches to data analytics. Coloring on simplicial complexes and Sperner type lemma. Fibrations, fibre bundles, vector bundles, principal G-bundles and classifying spaces. Micro-bundles, duality and Poincar\'e complexes. Simplicial fibre bundle theory and delta-homology of digraphs. (Co)homology spectral sequences, and double complexes.

Configuration spaces, iterated loop spaces and operads. H-spaces, Hopf invariants and Kervaire invariants. Cobordisms and topological quantum field theory. Lusternik-Schnirelmann category and topological complexity of motion planning. Homotopy theory of finite complexes. Toric topology and polyhedral products.

Prerequisite: General algebra, Linear algebra, pointed set topology.

References:

Curtis, E. B., Simplicial homotopy theory, Adv. Math., 6(2) (1971), 107-209.

Carlsson, G., Topology and Data, Bulletin AMS, 46 (2) (2009), 255-308.

Grigor'yan, A., Lin, Y., Muranov, Y., Yau, S.-T., Cohomology of digraphs and (undirected) graphs, Asian J. Math., 19(5) (2015), 887–931.

Grigor'yan, A., Lin, Y., Muranov, Y., Yau, S.-T., Homologies of path complexes and digraphs, Math arXiv: 1207.2834v4, 2013.

Grigor'yan, A., Muranov, Y., Yau, S.-T., On a cohomology of digraphs and Hochschild cohomology, J. Homotopy Relat. Struct., 11(2) (2016), 209-230.
Hatcher, A., Algebraic topology, Cambridge University Press, 2002.
James R. Munkres, Elements Of Algebraic Topology, Published January 1, 1996 by CRC Press, 468 pp. ISBN 9780201627282
Robert M. Switzer, Algebraic Topology-Homology and Homotopy, Springer-Verlag Berlin Heiselberg 2002,
George W. Whitehead, Elements of Homotopy Theory, GTM Vol. 61, Springer New York, NY, 746 pp. DOI https://doi.org/10.1007/978-1-4612-6318-0
Wu, J., Simplicial objects and homotopy groups//Braids: introductory lectures on braids, configurations and their applications, World Scientific Publishing Company, 2010, 31-181.

3. Title: Introduction to geometric representation theory

主讲人 Speaker: 李鹏辉(YMSC)

Course description: This is a course on representation theory with the point of view toward geometric representation theory. Topics include representation theory of semisimple algebraic group, Borel-Wei-Bott theorem, Beillinson-Berstein localization, Springer theory, Hecke algebras. If time permits, we shall also discuss further topics including categorification, character sheaves, etc.

Prerequisites: Abstract algebra, basic representation theory

Reference:

1. Neil Chriss, Victor Ginzburg - Representation Theory and Complex Geometry.
2. Ryoshi Hotta, Kiyoshi Takeuchi, Toshiyuki Tanisaki - D-Modules, Perverse Sheaves, and Representation Theory.

4. Title: 冯诺依曼代数

主讲人 Speaker: 吴劲松 (BIMSA)

Course description:

第一次课 算子连续函数演算

回顾算子的谱理论并简单介绍有界自伴算子上连续函数的运算规则。

第二次课 算子可测函数演算

简单介绍有界正规算子上 Borel 可测函数的运算规则。

第三次课 Gelfand 表示定理

简单介绍正线性泛函基本性质，给出 Gelfand-Naimark-Segal 构造，并由此说明 C*代数的忠实表示。

第四次课 冯诺依曼代数双换位定理

简单介绍 Hilbert 空间上算子拓扑，冯诺依曼代数定义及其双换位定理等。

第五次课 投影比较理论

简单介绍冯诺依曼代数中投影的比较关系理论。

第六次课 非交换 Lp 空间

简单讨论有界算子组成代数的非交换 Lp 空间的结构和性质。

第七次课 自由群冯诺依曼代数

简单讨论由离散群给出的冯诺依曼代数的性质。

第八次课 超有限冯诺依曼代数

简单讨论超有限冯诺依曼代数的不同类型。

第九次课 完全正映射

简单介绍完全正映射的基本性质。

预备知识：泛函分析

参考文献

Kadison, Ringrose, Fundamentals of the theory of operator algebras, I, II

Takesaki, Theory of operator algebras, I, II, III.

5. Title Introduction to p-adic linear differential equations

主讲人 Speaker: Tinhinane Amina Azzouz(BIMSA)

Course description: In the ultrametric setting, linear differential equations present phenomena that do not appear over the complex field. Indeed, the solutions of such equations may fail to converge everywhere, even without the presence of poles. This leads to a non-trivial notion of the radius of convergence, and its knowledge permits to obtain several interesting information about the equation. Notably, it controls the finite dimensionality of the de Rham cohomology. In this lecture we introduce the framework of p-adic differential equation. The course is divided into the following 9 lectures:

p-adic fields and analysis over them;

Lecture 1: The fields of p-adic numbers;
Lecture 2: Some ultrametric functional analysis;Differential algebra;
Lecture 3: Formalism of differential algebra;
Lecture 4: Metric properties of differential modules;
Lecture 5: Differential polynomials; The radii of convergence;
Lecture 6: First properties and decomposition with respect to the small radii of convergence;
Lecture 7: Behaviors of the radii of convergence under Frobenius pullback and push-forward;
Lecture 8: The decomposition theorem with respect to the radii of convergence; Areas of applications.
Lecture 9: An overview of some areas of applications.
Prerequisite: General algebra, Linear algebra, Galois theory of fields, Metric spaces.
References
G. Christol. “Le théorème de turritin p-adique (version du 11/06/2011)”.
G. Christol. “Modules différentiels et équations différentielles p-adiques”.In: Queen’s Papers in Pure and Applied Math (1983).
G. Christol and P. Robba. Équations différentielles p-adiques - Applications aux sommes exponentielles. Actualités Mathématiques. Hermann, 1994.
K. S. Kedlaya. p-adic differential equations. English. Cambridge: Cam- bridge University Press, 2010, pp. xvii + 380. isbn: 978-0-521-76879- 5/hbk.

6. Title: 多粒子模型简介

主讲人 Speaker: 蔚辉(YMSC)

课程简介：

自然界中小到细菌，大到鱼类、鸟类，乃至抽象层面人群的意识和决策等都有一个共同特点：个体之间简单交互，群体呈现出丰富多样的规律性行为。特别是集群智能行为，动植物群体会根据食物资源、狩猎者等障碍物，快速自发的形成群体层面较高效的运动模式。如何使用数学模型表述这一行为，用于研究个体之间的交互方式和运动规律，进行人工智能集群的仿生科技应用是一个正在不断发展的研究领域。本课程将介绍多粒子系统在微观尺度下的 Vicsek 模型、Kuramoto 模型等几类重要的常微分方程组模型，讨论其建模思想，以及相关的

开放性问题。

参考文献：

Juan A. Acebrón, L. L. Bonilla, Conrad J. Pérez Vicente, Félix Ritort, and Renato Spigler. The Kuramoto model: a simple paradigm for synchronization phenomena. *Rev. Mod. Phys.* 77, 137, 2005.

Tamás Vicsek, Anna Zafeiris. Collective motion, *Physics Reports,* Volume 517, Issues 3–4, Pages 71-140, 2012.

预备知识： 微积分，线性代数，可以使用 Matlab 等软件实现简单的计算和绘图。

7. Title: 速度和精度的学问——计算数学纵览

Speed and Accuracy – An Introduction to Computational Mathematics

主讲人 Speaker: 王珺(YMSC)

课程描述 Course Description：

计算数学自诞生起，一直是一门在争议中成长的学科。从最初的饱受争议，到今日发展壮大成庞大的"计算科学"体系，它经历了怎样的发展历程？从古老的牛顿消元到"时髦"的机器学习，有哪些重要的算法、思想在其间熠熠生辉？学习了线性代数，你或许惊艳于 Cramer 法则的简洁优美，为何时至今日计算数学家们仍对线性方程组穷究不舍？学习了微积分，你可曾对一众特殊函数情有独钟？可曾想过它们如何计算，又如何被应用？你对特征值问题的解法稔熟于心，可曾想过它与音乐有着千丝万缕的联系？著名计算数学家 Nick Trefethen 说："好的数值算法，应该在 10 秒内给出 10 位有效数字，所需的代码不应该超过一页纸。"当今的数值计算，真能做到这般极致吗？或许学完这门课，你可以尝试给出自己的见解。

本短期课程以数学类专业本科生为主要授课对象。以"精度"与"速度"为主线，以同学们熟知的微积分和线性代数中的经典问题为切入点，带领同学们初步探索数值计算的世界。用简洁而富有挑战性的例子，向同学们展现丰富而又深刻的数值计算的世界。让同学们领略到数学"经世致用"的"另一面"。所涉及内容包括：逼近论、渐近分析、快速算法、数值线性代数、图像处理、微分方程数值解等。

课程以课堂讲述和上机实践相结合。课程期间鼓励学生完成至少一个科学计算程序的开发。

课程拟安排 1-2 节计算数学领域的前沿研究报告。

预备知识 Prerequisites：必修微积分、线性代数、程序设计初步知识（建议语言：C/C++, Fortran, Python, MATLAB, Julia 任选其一）。

课程中所涉及的较深入内容将以补充阅读材料的形式给出。

参考资料 References：本课程不使用固定教科书。以课堂讲义和研究论文为主要资料。