1 Mini-course information

- **Course title:** Quantitative Differentiation on Spaces with Lower Curvature Bounds
- **Instructor:** Nan Li (The City University of New York)
- **Course Schedule:**

Date	Day	Session	Time
December 11, 2025	Thursday	**Lecture 1**	2pm - 5pm
December 13, 2025	Saturday	**Lecture 2**	2pm - 5pm
December 14, 2025	Sunday	**Lecture 3**	2pm - 5pm

- **Course Description:** Quantitative differentiation is a technique initiated by Jeff Cheeger and Aaron Naber in 2013. The core idea of this technique is to establish a controlled quantitative stratification structure based on two fundamental components: a monotonic formula and an almost rigidity property. Due to the fundamental nature of its starting points, this technique has been successfully applied to many areas of geometric analysis, including spaces with bounded Ricci curvature, harmonic maps, and Ricci flow, leading to numerous deep results.
  In this mini-course, we will discuss the applications of this technique to manifolds with lower sectional curvature bounds (and their generalization Alexandrov spaces). This can be viewed as a model case for understanding the method's core mechanics.

  Topics will include:

  1. Starting points: The monotonic formulas on Alexandrov spaces
  2. The annulus covering theorem and an application: packing estimates of singular sets
  3. Another application: bounding the curvature integral

2 Further reading references

Foundational Texts

1. D. Burago, Y. Burago, and S. Ivanov, *A Course in Metric Geometry*, American Mathematical Society, 2001.
2. Y. Burago, M. Gromov, and G. Perelman, "A.D. Alexandrov spaces with curvature bounded below," *Russian Mathematical Surveys*, vol. 47, no. 2, pp. 1–58, 1992.

On Spaces with Lower Sectional Curvature Bound & Alexandrov Spaces

3. N. Li and A. Naber, "Quantitative estimates on the singular sets of Alexandrov spaces," *Inventiones Mathematicae*, vol. 224, pp. 313–373, 2021.
4. N. Li, "Aspects and examples on quantitative stratification with lower curvature bound," in *Differential Geometry in the Large*, London Mathematical Society Lecture Note Series, Cambridge University Press, 2020.

5. A. Petrunin, "An upper bound for the curvature integral," *St. Petersburg Mathematical Journal*, vol. 20, no. 5, pp. 799–806, 2009.

6. N. Li, "An alternative proof for the upper bound of curvature integral on manifolds with lower sectional curvature bound," *Journal of Mathematical Study*, 2025.

On Manifolds with Lower Ricci Curvature Bound

7. J. Cheeger and T. H. Colding, "On the structure of spaces with Ricci curvature bounded below I," *Journal of Differential Geometry*, vol. 46, no. 3, pp. 406–480, 1997.

8. J. Cheeger and T. H. Colding, "On the structure of spaces with Ricci curvature bounded below II," *Journal of Differential Geometry*, vol. 54, no. 1, pp. 13–35, 2000.

9. J. Cheeger and T. H. Colding, "On the structure of spaces with Ricci curvature bounded below III," *Journal of Differential Geometry*, vol. 54, no. 1, pp. 37–74, 2000.

10. J. Cheeger and A. Naber, "Lower bounds on Ricci curvature and quantitative behavior of singular sets," *Inventiones Mathematicae*, vol. 197, no. 2, pp. 405–457, 2014.

11. J. Cheeger and A. Naber, "Regularity of Einstein manifolds and the codimension 4 conjecture," *Annals of Mathematics*, vol. 182, no. 3, pp. 1093–1165, 2015.

12. J. Cheeger, W. Jiang, and A. Naber, "Rectifiability of singular sets in noncollapsed spaces," *Annals of Mathematics*, vol. 193, no. 2, pp. 407–538, 2021.

Other Settings

13. J. Cheeger and A. Naber, "Quantitative stratification and the regularity of harmonic maps and minimal currents," *Communications on Pure and Applied Mathematics*, vol. 66, no. 6, pp. 965–990, 2013.

14. J. Cheeger, A. Naber, and D. Valtorta, "Critical sets of elliptic equations," *Communications on Pure and Applied Mathematics*, vol. 68, no. 2, pp. 173–209, 2015.

15. A. Naber and D. Valtorta, "Rectifiable-Reifenberg and the regularity of stationary and minimizing harmonic maps," *Acta Mathematica*, vol. 224, no. 1, pp. 1–78, 2020.

16. J. Cheeger, R. Haslhofer, and A. Naber, "Quantitative stratification and the regularity of mean curvature flow," *Geometric and Functional Analysis*, vol. 27, no. 4, pp. 932–971, 2017.

17. M. Focardi, A. Marchese, and E. Spadaro, "Improved estimate of the singular set of Dir-minimizing Q-valued functions via an abstract regularity result," *Analysis & PDE*, vol. 11, no. 6, pp. 1397–1452, 2018.

18. A. Naber and D. Valtorta, "The singular structure and regularity of stationary and minimizing varifolds," *Annals of Mathematics*, vol. 190, no. 2, pp. 413–565, 2019.

Open Problems

19. M. Gromov, "Large Riemannian manifolds," in *Lecture Notes in Mathematics*, vol. 1201, Springer, 1986.

20. S.-T. Yau, "Open problems in geometry," in *Chern—A Great Geometer of the Twentieth Century*, International Press, 1992.

21. A. Naber, "Conjectures and open questions on the structure and regularity of spaces with lower Ricci curvature bounds," *Journal of Differential Geometry*, vol. 113, no. 1, pp. 1–40, 2019.