YMSC COURSE: CREATIVE PROBLEM SOLVING, QIUZHEN COLLEGE, TSINGHUA UNIVERSITY, FALL 2024

COURSE SYLLABUS

1. General information

Instructor: Cezar Lupu (Assistant Professor BIMSA)
Teaching Assistant-Homework Grader: Cezar Lupu
Office: Shuanqing Complex Building (Cezar Lupu)
Email: lupucezar@bimsa.cn (Cezar Lupu)
Lecture Time:

- Thursday (Lecture), 19:20-20:55 PM
- Friday (Recitation), 19:20-20:55 PM

Location: (Shuanqing Complex Building) B626
Office hours: Make an appointment with the instructors via email. The timetable meetings are given below.

- Thursday, 18:00-19:00 PM
- Friday, 18:00-19:00 PM

Credits: no credit units
Topics covered: Elementary Algebra, Abstract & Linear Algebra, Real Analysis, Combinatorics, Geometry & Trigonometry, Number Theory, Probability, Differential Equations

2. Course description

This course (YMSC-Creative Problem Solving) teaches important skills in problem solving that are not taught in a systematic way in any other course. These skills are extremely valuable in preparing students for jobs and for graduate-level research. The teaching style will be a mixture of a lecture and a problem-solving

session. Training will involve the study of problems from previous Putnam competitions, for which this course can be regarded as a useful preparation. An attempt will be made to look for unifying mathematical ideas. General strategies for solving problems will also be discussed.

By the end of this course, students should develop fundamental problem solving skills, and become accustomed to concentrating on a problem for an extended period of time. This course will be focused on the raw creative problem-solving skills which can serve as an essential ingredient in almost every field of activity.

Last but not least, after students take the Shadow Putnam competition, they will have the chance to experience some research oriented talks suitable for undergraduates from various faculty at Tsinghua.

2.1. **The 3rd Shadow Putnam Competition for students at the Qiuzhen College, Tsinghua University.** The Shadow Putnam Mathematical Competition is a test that students from Qiuzhen College take in the second Sunday of December. The problems are the same as the ones from The William Lowell Putnam Mathematical Competition which is the premiere competition for undergraduate students in North America. More than 500 universities compete in this contest organized by the Mathematical Association of America (MAA).

The test is supervised by faculty members of each participating university. Every problem is graded on a scale of 0-10. The problems are usually listed in increasing order of difficulty, with A1 and B1 the easiest, and A6 and B6 the hardest. Top 5 scoring students on the Putnam exam are named Putnam Fellows. A student can take this exam maximum four times and the Putnam official team of the university consists of 3 members.

The 3rd Shadow Putnam Mathematical Competition, 2024

It will take place on **SUNDAY, DECEMBER 8TH** and it consist of two sessions of three hours each:

Morning Session: 8:00-11:00 AM, location: TBD.
Afternoon Session: 13:00-16:00 PM, location: TBD.

3. Grading Policy, Grading Scale, Weighted Value of Assignments and Tests

- Homework assignments: **60%** (4 homework assignments in total; you must solve 10-15 problems from each homework to get full credit!)
- Participation in the Putnam competition: **20%** (you must take both sessions of the exam!)
- Performance in the Putnam competition: **20%** (if the participant's score is greater or equal than 30 points which is 3 problems completely solved)

Letter grades will then be assigned in accordance with the following correspondence:

- Letter **grade A** = a percentile grade of 90% of higher
- Letter **grade A-** = a percentile grade of 80% or higher, that is lower than 90%
- Letter **B**= a percentile grade of 70% or higher, that is lower than 80%
- Letter **B-**=a percentile grade of 60% or higher, that is lower than 70%
- Letter **C**=a percentile grade lower than 60%

4. Tentative Schedule of Class Meetings and Topics

Week 1: Elementary algebra (September 12, 13)

This will cover problems on topics such as algebraic identities and inequalities as well as complex numbers, mathematical induction, functional equations and polynomials (integer polynomials, roots of polynomials).

Week 2: Geometry and trigonometry (September 19, 20)

This will cover problems on topics such as vectors, conics, quadratics, and other curves in the plane as well as trigonometric formulas.

Week 3: Combinatorics (September 26, 27)

This will cover problems on topics combinatorial geometry, pigeonhole principle, generating functions graph theory, binomial identities and counting strategies.

Week 4: No classes (National day holiday) (October 3, 4)

Week 5: Probability (October 10, 11)

This will cover problems on continuous random variables. Jointly continuous random variables, independence, conditioning, functions of one or more random variables, change of variables. Examples including some with later applications in statistics.

Week 6: Number Theory I (October 17, 18)

This will cover problems on topics such as integer-valued sequences and functions, congruences, divisibility and arithmetic functions.

Week 7: Number theory II (October 24, 25)

This will cover problems on topics such as quadratic residues, diophantine equations, and algebraic & analytic methods in number theory.

Week 8: Abstract algebra (October 31, November 1)

This will cover problems on topics such as groups, rings, and finite fields.

Week 9: Linear algebra I (November 7, 8)

This will cover topics on 2×2 and 3×3 matrices and determinants. Also, will cover problems on topics such as vectors spaces, linear transformations, characteristic and minimal polynomials, eigenvalues and eigenvectors.

Week 10: Linear algebra II (November 14, 15)

This will cover some special topics in linear algebra such as special classes of diagonalizable matrices, Jordan canonical form, Schur triangularization form, spectral theorem for normal matrices.

Week 11: Real analysis I (November 21, 22)

This will cover problems on topics such as sequences and series of real numbers. Moreover, we will also cover problems on topics such as limits of functions and continuity.

Week 12: Real analysis II (November 28, 29)

This will cover problems on topics such as differentiability (mean value theorems, Taylor series, etc) and integrability (Riemann integrals, continuity of integrals, integral inequalities). Also, we plan to cover sequences and series of functions.

Week 13: Real analysis III (December 5, 6)

This will cover problems on multivariable differential and integral calculus. This will include extrema of functions, Lagrange multipliers, Fubini's theorem, change of variables formulas and divergence theorem.

References

[1] R. Gelca, T. Andreescu, *Putnam and Beyond*, Springer Verlag, 2007.
[2] K. Kedlaya, B. Poonen, R. Vakil, *The William Lowell Putnam Mathematical Competition 1985-2000: Problems, Solutions and Commentary*, The Mathematical Association of America, Washington, D.C., 2002.
[3] K. Kedlaya, The Putnam Archive (1985-2021).
[4] L. Larson, Problem-Solving Through Problems, Springer Verlag, 1983.