Abstract

While classical statistics has dealt with observations which are real numbers or elements of a real vector space, nowadays many statistical problems of high interest in the sciences deal with the analysis of data which consist of more complex objects, taking values in spaces which are naturally not (Euclidean) vector spaces but which still feature some geometric structure.

The manifold fitting problem can go back to H. Whitney's work in the early 1930s ([8]), and finally has been answered in recent years by C. Fefferman's works [1, 7]. The solution to the Whitney extension problem leads to new insights for data interpolation and inspires the formulation of the Geometric Whitney Problems ([3, 4]): Assume that we are given a set $\mathcal{Y} \subset \mathbb{R}^D$. When can we construct a smooth d-dimensional submanifold $\widehat{\mathcal{M}} \subset \mathbb{R}^D$ to approximate $\mathcal{Y}$, and how well can $\widehat{\mathcal{M}}$ estimate $\mathcal{Y}$ in terms of distance and smoothness?

To address these problems, various mathematical approaches have been proposed (see [3, 4, 6, 2, 5]). However, many of these methods rely on restrictive assumptions, making implementing them as efficient algorithms challenging. As the manifold hypothesis (non-Euclidean structure) continues to be a foundational element in statistics, the manifold fitting problem, merits further exploration and discussion within the modern statistical community. The talk will be partially based on some recent works [10, 9] along with some on-going progress.

References

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