Exercises

1. Compute the Kauffman brackets and Jones polynomials of the following links:
   a. Right and left Hopf links;
   b. Right and left trefoil knots;
   c. The eight-figure knot;
   d. The Whitehead link;
   e. The Borromean rings.

2. Let D' be a diagram which is obtained from an orientable diagram D by applying one increasing Reidemeister move Ω_1. Show that $\langle D' \rangle = (-a)^{\pm 3}\langle D \rangle$.

3. Show that the Kauffman bracket of a link's mirror image is the Kauffman bracket of the initial link with a replaced by a^{-1}.

4. Prove the following relation for the Jones polynomial:

$$a^{-4}X(L_+) - a^4X(L_-) = (a^2 - a^{-2})X(L_0).$$

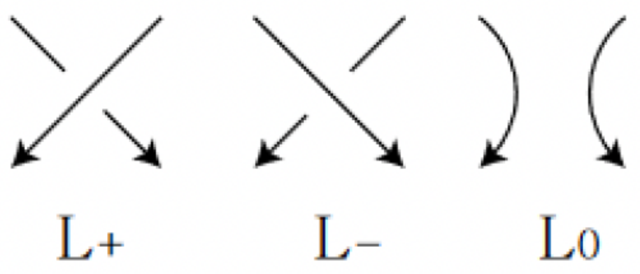


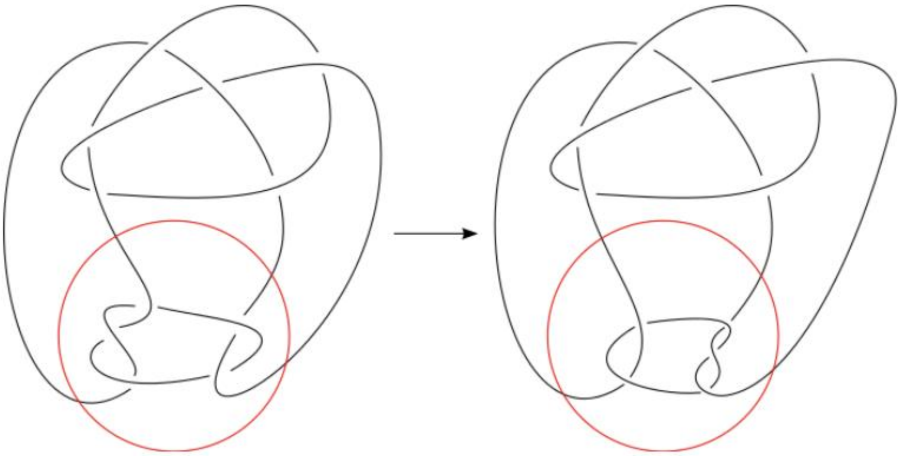
Fig. 2: Diagrams of skein-relations

5. Prove that the Jones polynomial does not detect knot invertability.

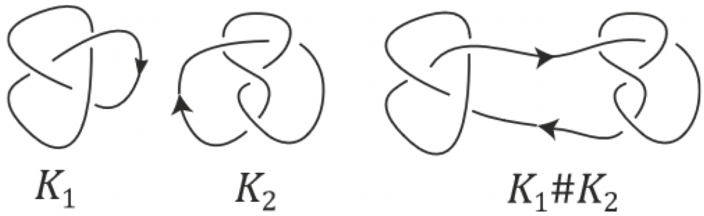
6. Prove that the equality $X(K_1 \sqcup K_2) = (-a^2 - a^{-2})X(K_1)X(K_2)$ holds for the Jones polynomial of a disjoint union $K_1 \sqcup K_2$.

7. Prove that the equality $X(K_1 \# K_2) = (-a^2 - a^{-2})X(K_1)X(K_2)$ holds for the Jones polynomial of a connected link sum $K_1 \# K_2$.

8. Prove that the Kauffman bracket cannot distinguish mutant knots.



9. Prove that the length of a connected diagram does not exceed 4n, but the alternating one without splitting points is equal to 4n.
Prove that the alternating diagrams without splitting points are minimal.
10. Evaluate the length of the Kauffman bracket for k-fold cable of a link diagram with n components.
11*. Prove the minimality of adequate diagrams by only using the Jones polynomial (Estimate the length of the Jones polynomial of the satellite knot).
12. Prove that the Kauffman bracket never identically equals to 0.
13.. Show that the diagram of a knot $K_1 \# K_2$ (Fig.1) isn't alternating, however it is adequate.
Fig. 1: Connected sum $K_1 \# K_2$ of the oriented knot-diagrams



Problems

1. Does the Jones polynomial detect the unknot?
2. How to extend the Kauffman bracket to coloured links.
Consider a knot colouring (for example, Fox colouring or some other one).
Then for each crossing define the skein-relation depending on colourings as shown in Fig.3
(With the coefficients A_{x,y} etc. depending on the colours a,b); define the loop value also depending on some
colours.
How to find the coefficients A_{x,y}, \cdots in order to get a knot invariant?
The obvious solution is when we put A_{x,y} = a, B_{x,y} = a^{-1} regardless the colours (the loop value is set to be (-a^{2}-a^{-2}).
How to make such invariants stronger if we put colours into account?

There was a successful research due to Nelson, Orrison, and Rivera when a certain sum was taken over all colourings
(so they got an invariant of the initial knot by using coloured knots).

Fig. 3: Colored bracket