《Riemann假设与素数分布》

国家重点研发计划项目推进会暨数论前沿研讨会

■ 2025-03-21 ~ 2025-03-23

清華大學 丘成桐数学科学中心

Yau Mathematical Sciences Center, Tsinghua University

Organizers

- Hansheng Diao (刁晗生, YMSC)
- Lei Fu (扶磊, YMSC)
- Yueke Hu (胡悦科, YMSC)
- Bin Xu (徐斌, YMSC)

Location

- 北京市怀柔区雁栖湖应用数学研究院A7-101
- Venue Online ID Password
- A7-101 Zoom 442 374 5045 BIMSA

住宿

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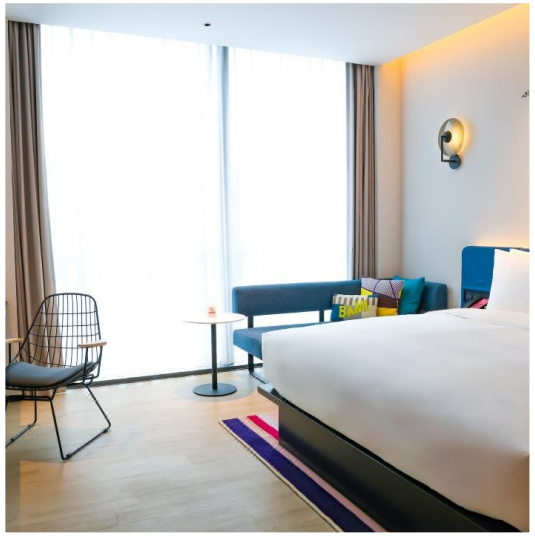

用餐

03/21-03/23 Breakfast
雅乐轩酒店一层（F1, Aloft Hotel）

03/21-03/23 Lunch
A4，Bimsa

会议议程

	Mar 21 Fri	Mar 22 Sat	Mar 23 Sun
08:20-08:30	Opening Ceremony	/	/
08:30-09:30	J. Liu	H. Diao	Z. Li
09:40-10:40	H. Yu	J. Xie	Z. Tian
11:00-12:00	W. Zheng	P. Xi	Z. Sun
13:30-14:30	Y. Hu	M. Chen	B. Huang
14:40-15:40	Y. Ding	X. Shen	H. Qin
16:00-17:00	S. Li	X. Yuan	

报告内容

Jianya Liu（ 刘建亚 ，Shandong University ）

轨道上的素数分布

素数分布是数论研究的核心领域，而多样化的分析方法是探索素数问题的重要途径。Sarnak提出的群作用轨道上素数分布的纲领性猜想，不仅涵盖了哥德巴赫猜想等著名难题，还预测了素数在非线性、非交换新场景下的分布规律。这一领域堪称现代数学思想的实验田，分析、代数、几何、组合等思想方法在此高度交叉融合。本演讲将回顾Sarnak猜想的研究历史与思想方法，并重点介绍其最新研究进展。

报告内容

Hongjie Yu（余红杰，MCM, CAS）

Dimensions of Cusp Forms over Function Fields and Hitchin Moduli Spaces

The dimension formula is a classical result in the theory of modular forms. We are interested in an analogous question over a function field for cusp forms of split reductive groups. In the case of the general linear group, this question is connected to Deligne's conjectures on counting l-adic local systems. However, in this setting, our understanding of the residual spectrum within the full automorphic spectrum remains incomplete. To circumvent this difficulty, we can consider prescribed generic ramifications at certain places. In this talk, I will present some results showing that the dimension is expressed in terms of the number of F_q-points of Hitchin moduli spaces.

报告内容

Weizhe Zheng（郑维喆，MCM, CAS）

Arithmetic properties of l-adic cohomology over p-adic fields

I will give an overview of arithmetic properties of l-adic cohomology of algebraic varieties and rigid spaces over p-adic fields. A special focus will be given to integrality and p-adic valuations of Frobenius eigenvalues, with applications to weights and quasi-unipotence. This talk is partly based on joint work with Qing Lu.

报告内容

Yongquan Hu（胡永泉，MCM, CAS）

Finite length results for the mod p cohomology of GL_2

In the mod p Langlands program for GL_2, it is important to study the Hecke eigenspace of mod p cohomology of Shimura curves. Inspired by the work of Breuil-Paskunas, it is conjectured that such representations have finite length and a special shape. In this talk, I will explain the proof of the (expected) upper bound of the length under some reasonable hypotheses. This is joint work with Breuil, Herzig, Morra and Schraen.

报告内容

Yiwen Ding (丁一文 , Peking University)

Hodge filtration and p-adic Langlands program

A basic and initializing problem in p-adic Langlands program is to recover the information of the Hodge filtration of a p-adic Galois representation on the automorphic side. In this talk, we will first discuss the pioneering case of $GL_2(\mathbb{Q}_p)$, which remains the only fully understood case in the p-adic Langlands program for nearly 20 years. Then we discuss some conjectures and recent progresses towards the problem in the general case.

报告内容

Shizhang Li（李时璋，MCM, CAS）

Cohomology of p-adic local systems and duality

The cohomology of a rational p-adic local system on a smooth proper rigid space was known to not have finiteness in a naïve sense. Yet a theorem of Kedlaya--Liu showed that these cohomologies are always Banach--Colmez spaces, which can also be viewed as some kind of finiteness. Recently Anschütz--Le Bras--Mann find a new proof of this theorem as well as establish a version of Poincaré duality, by using heavy machines of 6 functor formalisms of v-sheaves that they developed. In this talk, we shall explain a proof of these facts by constructing natural maps and chasing only 1 (but huge) diagram. This is a joint work in progress with Wiesława Nizioł, Emanuel Reinecke, and Bogdan Zavyalov.

报告内容

Hansheng Diao（刁晗生，YMSC）

Rigidity of crystalline local systems

We study certain rigidity properties of p-adic local systems on a smooth scheme X over a p-adic field. In particular, we show that the monodromy of the log (iso)crystal attached to a semistable local system is rigid along irreducible components of the special fiber. There are several applications. Firstly, suppose that X has good reduction. We show that, if a family of semistable representations is crystalline at one classical point on X, then it is crystalline everywhere. Secondly, for any p-adic local system on a smooth projective curve with good reduction, if it is potentially crystalline at one classical point, then it is potentially crystalline everywhere. Finally, we show that a p-adic local system over the punctured disc extends to a (necessarily crystalline) p-adic local system over the entire disc if it is crystalline at all classical points. In other words, such a local system cannot have geometric monodromy if it has no arithmetic monodromy everywhere on the punctured disc. This is a joint work with Zijian Yao.

报告内容

Junyi Xie（谢俊逸，Peking University）

Numerical spectrums control Cohomological spectrums

Let f be an endomorphism of projective variety. We show that the spectral radius of the pull-back of f on the numerical groups of codimension i and the l-adic cohomology group of degree 2i are the same. As a consequence, if f is q-polarized for some q>1, we show that every eigenvalue of the pull-back of f on the j-th cohomology group is qj/2. This generalizes Delignes's theorem for Weil's Riemann Hypothesis to arbitrary polarized endomorphisms and proves a conjecture of Tate.

报告内容

Ping Xi (郗平 , Xi'an Jiaotong University)

Sign changes of Hecke eigenvalues at squares

Given a holomorphic Hecke cusp form f for the full modular group, Matomaki--Radziwill proved that the first Fourier coefficients of f change signs at $\gg$ n positive integers. They also proved the number of sign changes can be $\gg$ n/log n in the situation of Hecke--Maass cusp forms. However, their arguments do not apply to symmetric powers (self-dual forms for higher-rank groups). In this talk, we show that the number of sign changes up to n can be $\gg n^{1-O(1)}$ in the situation of symmetric squares. This is an ongoing work joint with Junren Zheng.

报告内容

Miaofen Chen（陈苗芬，Fudan University）

Monodromy and connected components of the moduli spaces of p-adic Shtukas in the HN-decomposable case

Gleason, Lim and Xu determine the set of connected components of affine Deligne-Lusztig varieties via the moduli spaces of p-adic Shtukas in the HN-irreducible case. Motived by their work, in this talk, we want to discuss the set of connected components of the moduli spaces of p-adic Shtukas in the HN-decomposable case and its relation to affine Deligne-Lusztig varieties.

报告内容

Xu Shen（申旭，MCM, CAS）

Bruhat-Tits buildings and p-adic period domains

Bruhat-Tits buildings and p-adic period domains are both basic objects associated to p-adic reductive groups. In this talk, we will discuss some basic relations between them. This is joint work with Ruishen Zhao.

报告内容

Xinyi Yuan（袁新意，Peking University）

Bounding Numbers of Rational Points on Curves

The Mordell conjecture proved by Faltings asserts that there are only finitely many rational points on a curves of genus greater than 1. The uniform Mordell problem asks for upper bounds of the number of rational points. In this talk, we introduce various conjectures and theorems concerning the uniform Mordell problem.

报告内容

Zhiyuan Li（李志远，Fudan University）

Theta series and Shimura varieties of orthogonal type

In this talk, I will explore the fascinating interplay between lattice theory and vector-valued modular forms via theta series, presenting an elegant connection that bridges these areas. I will also discuss its applications on the study of the Picard group of Shimura varieties. Our findings reveal that the Picard group of the Baily-Borel compactification of a broad class of Shimura varieties is isomorphic to $\mathbb{Z}$.

报告内容

Zhiyu Tian（田志宇，Peking University）

Kato homology for rationally connected fibrations

Kato homology is the homology of a Gersten type complex, first studied by Kato and Bloch-Ogus, and measures the difference between algebraic cycles and a homology theory. Motivated by the study of zero-cycles of a rationally connected variety over a local or global field, and conjectures of Manin-Peyre, Cohen-Jones-Segal, Schreieder, Voisin, etc., I will explain a set of conjectures of the Kato homology of a rationally connected fibration, and some evidences.

报告内容

Zhiwei Sun（孙智伟，Nanjing University）

On infinite series with summands involving binomial coefficients

In this talk, we introduce the developments of infinite series with summands involving binomial coefficients. In particular, we will mention some recent results and conjectures of the speaker including the fastest converging series for computing $\zeta(5)$.

报告内容

Bingrong Huang（黄炳荣，Shandong University）

模形式的值分布与 L-函数

本报告，我们将探讨模形式值分布的一些结果。我们首先介绍 Hecke 特征型的全纯量子唯一遍历定理和去相关性定理。作为推论，我们可以得到一些模形式的零点分布结果。然后，我们将讨论 Hecke 特征型的联合值分布。在广义黎曼假设和 Ramanujan 猜想下，我们可以证明一些值分布的条件性结果，并得到三重积 L-函数一阶矩的渐近公式。无假设条件下，我们可以证明一些期望和方差的结果。主要工具是 L-函数的矩的估计。

报告内容

Hourong Qin（秦厚荣，Nanjing University）

A Connection between the Milnor K_2 Group and the Shafarevich-Tate Group

Let $p \equiv 1 \pmod 8$ be a prime and E_p the elliptic curve defined by $y^2=x^3-p^2x$. Denote by O_F the ring of integers of $\mathbb{Q}(\sqrt{P})$. We establish a connection between the Milnor K_2 group of O_F and the Shafarevich-Tate group of E_p. Our results rely on investigations into quadratic forms, L-values of elliptic curves, and the interplay between ideal class groups and Milnor K_2 groups.

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