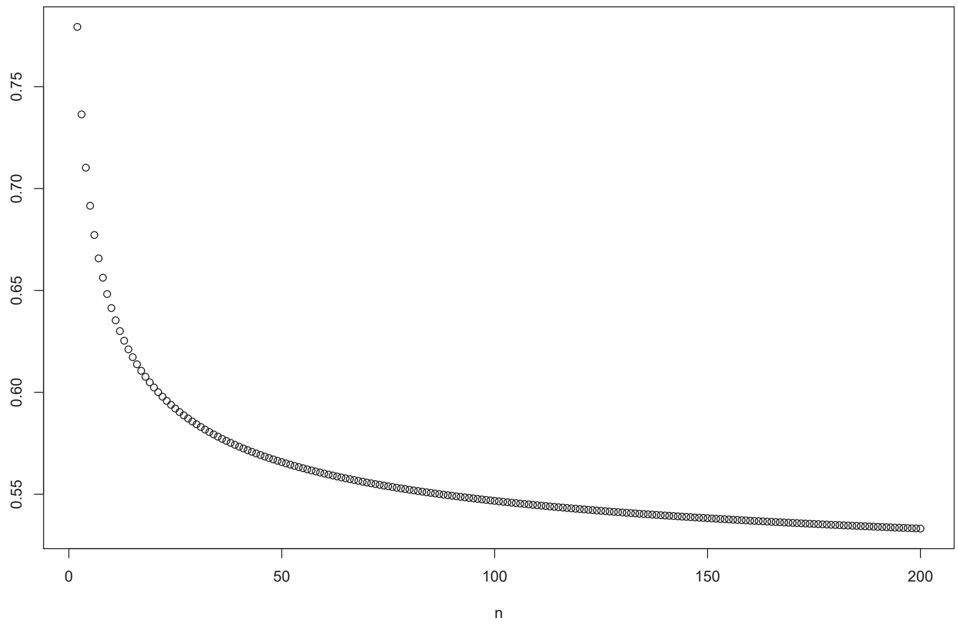


FIGURE 1. *Plot of* $\mathbb{P}(\chi^2_{n-1} < n-1/n)$ *as a function of* $n$, $n = 2, \ldots, 200$.

that is, $S^2_{\mathrm{U},n}$ and $S^2_{\mathrm{SV},n}$ are both asymptotically Pitman closer to $\sigma^2$ than $S^2_{\mathrm{M},n}$. It is therefore clear, at least on these Gaussian examples, that we should be cautious when choosing to shrink the variance for point estimation purposes. As we recently discovered, the examples in Yatracos [19] are similar to those studied in Khattree [7, 8, 9, 10].

In the present paper, we generalize this discussion to a large class of distributions. Taking a more general point of view, we let $X_1, \ldots, X_n$ be a sample drawn according to some unknown distribution with finite variance $\sigma^2$, and consider two candidates to estimate $\sigma^2$, namely

$$S^2_{1,n} = \alpha_n \sum_{i=1}^{n} (X_i - \bar{X}_n)^2 \quad \text{and} \quad S^2_{2,n} = \beta_n \sum_{i=1}^{n} (X_i - \bar{X}_n)^2 .$$

Assuming mild moment conditions on the sample distribution, our main result (Theorem 1) offers an asymptotic development of the form

$$\mathbb{P}\left(\left|S^2_{2,n} - \sigma^2\right| \geq \left|S^2_{1,n} - \sigma^2\right|\right) = \frac{1}{2} + \frac{\Delta}{\sqrt{n}} + \mathrm{o}\left(\frac{1}{\sqrt{n}}\right),$$

where the quantity $\Delta$ depends both on the moments of the distribution and the ratio of the sequences $(\alpha_n)_n$ and $(\beta_n)_n$. It is our belief that this probability should be reported in priority before deciding whether to use $S^2_{2,n}$ instead of $S^2_{1,n}$, depending on the sign and values of $\Delta$. Standard distribution examples together with classical variance estimates are discussed, and similar results pertaining to the estimation of the standard deviation $\sigma$ are also reported.


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