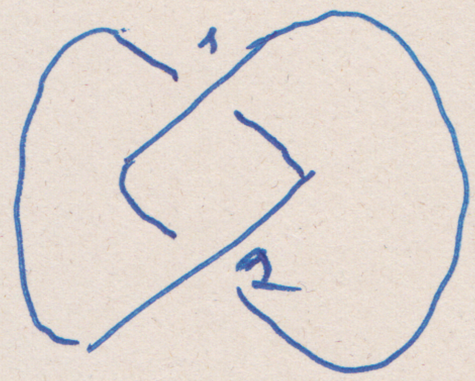
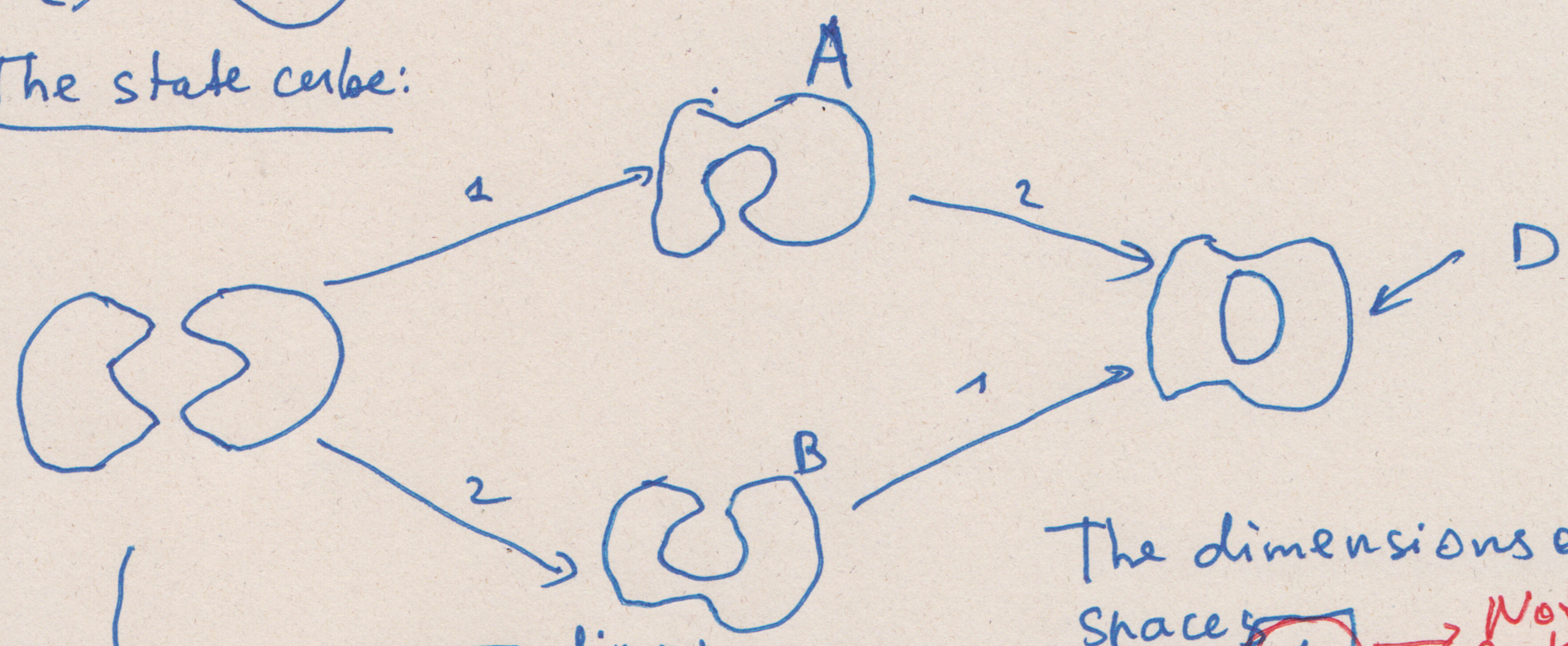
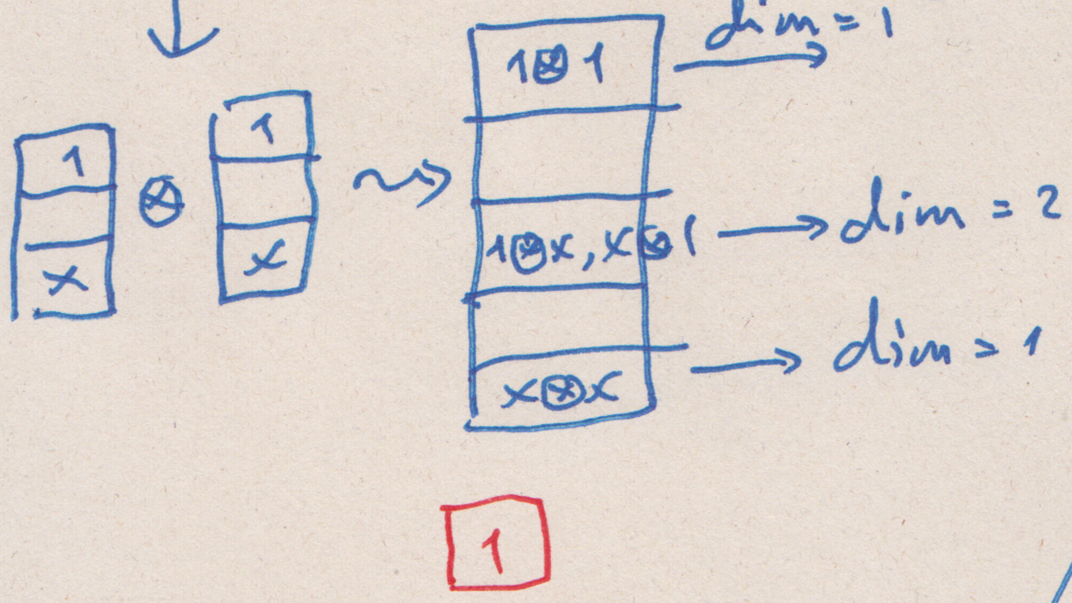
# Khovanov homology.

## The Hopf link.




**The state cube:**

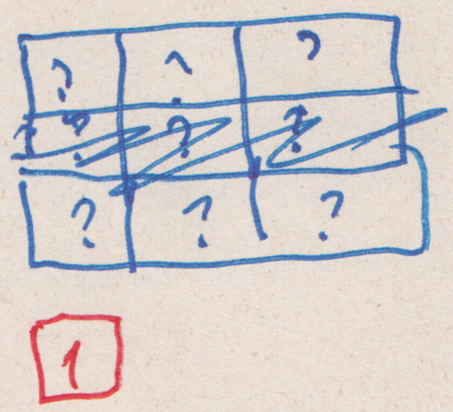



The dimensions of chains spaces:

| | | |
|---|---|---|
| | | 1 |
| | | |
| 1 | 2 | 2 |
| | | |
| 2 | 2 | 1 |
| | | |
| 1 | | |

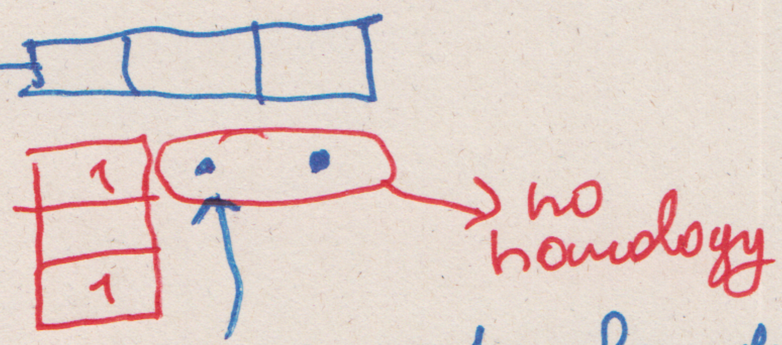
Nothing on the left; nothing on the right → A cycle which is not a boundary




$d(1\otimes X) = d(X\otimes 1) = X_A + X_B$

$\Downarrow$

$d(1\otimes X - X\otimes 1) = 0 \Rightarrow$ a cycle




$X_A$ is not a boundary

$X_A \to (\pm)(X\otimes X)_D$

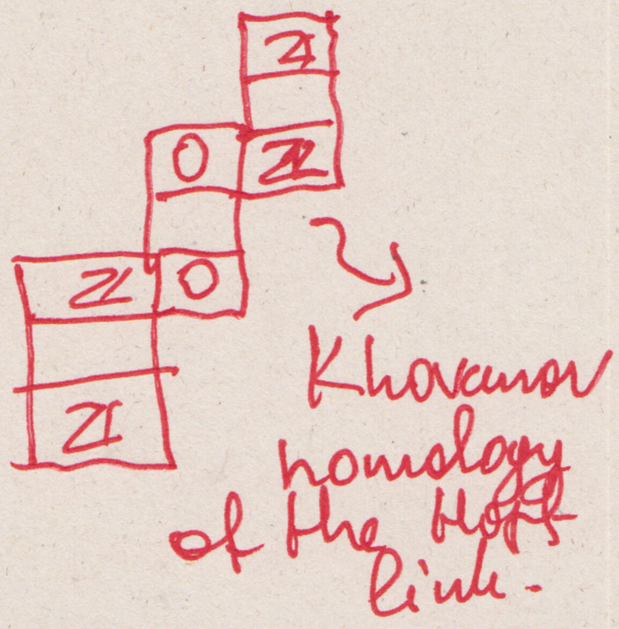
$1\otimes 1 \longrightarrow$

$d(1\otimes 1) = 1_A + 1_B \Rightarrow 1\otimes 1$ is not a cycle

$1_A$ is not a boundary $\Rightarrow$

$d(1_A) = \pm(1\otimes X + X\otimes 1)_D$

Now, $(1\otimes X)_D$ is a cycle which is not a boundary $\Rightarrow$




Khovanov homology of the Hopf link.