Observation in QM.

Axiom of observation:

phys. state – $\ell|\psi\rangle$ in proj. space

Observable – phys. Quantity (corresponding to the observation device) – $\mathcal{O}$

Observation is probabilistic, we get results λ_i with probability

$Tr(Proj_i \cdot Proj_{\ell|\psi\rangle})$, $Proj_i$ is the projector to the eigenspace, corresp. to eigenvalue λ_i of $\mathcal{O}$, $Proj_{\ell|\psi\rangle}$ is a projector to the line $\ell|\psi\rangle$.

How does it happen?

Example: $\mathbb{C}^2_{\setminus 0}/\mathbb{C}^* = \mathbb{CP}^1$

Observables – Herm. operators in $\mathbb{C}^2$

$T_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $T_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Let us take a state corresp. to $|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

It is an eigenstate of T_3, so we will get in observation $+1$ (cor. eigenvalue) with probability 1.

At the same time, if we will observe T_1, its eigenspaces are

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}_+ = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \leftarrow \text{eigenspaces}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}_- = -\begin{pmatrix} 1 \\ -1 \end{pmatrix} \leftarrow$$

So $\mathrm{Tr}(\mathrm{Proj}_+, \mathrm{Proj}_{\ell(\psi)}) = \frac{1}{2}$

$\mathrm{Tr}(\mathrm{Proj}_-, \mathrm{Proj}_{\ell|\psi\rangle}) = \frac{1}{2}$

probability that we will get in observation of T_1 in the state $|\psi\rangle$ $\;1$ is $\frac{1}{2}$

-1 is $\frac{1}{2}$

Thus: 1) Average in observation of T_i in state $|\psi\rangle$ is 0!

2) Distribution of the observations has a dispersion

Reminder:

If we have a distribution for a random quantity Q we have an average $\langle Q \rangle = \sum_i Q_i \omega_i$

i is what can happen, Q_i is the value of Q if event i happens

ω_i is the probability that event i happens.

Consider Q^2 - also also random quantity: $\langle Q^2 \rangle = \sum_i Q_i^2 \omega_i$

measure, showing how random Q is

If Q is not actually random, $\omega_1 = 1$, $\omega_i = 0$, $i \neq 1$, then $\langle Q^2 \rangle = (\langle Q \rangle)^2$, but in general, for random Q we have

(D) $\langle Q^2 \rangle - (\langle Q \rangle)^2 \geq 0$ dispersion of Q $D(Q)$

dispersion, not differential

consider new random variable

$$\tilde{Q} = \underline{Q - \langle Q \rangle}.$$

just a number

$$\langle -2Q \cdot \langle Q \rangle \rangle$$

$$\underset{0}{\langle \tilde{Q} \rangle = 0}, \quad \langle \tilde{Q}^2 \rangle = \langle Q^2 \rangle - \underline{2\langle Q \rangle \langle Q \rangle} + \langle Q \rangle \langle Q \rangle = \langle Q^2 \rangle - (\langle Q \rangle)^2$$

In particular, in the example considered above the dispersion of T_1 on the state corresponding to $\ell\binom{1}{0}$ is $\langle T_1^2 \rangle - (\langle T_1 \rangle)^2 =$

$= 1 - 0 = 1$ So dispersion is really big

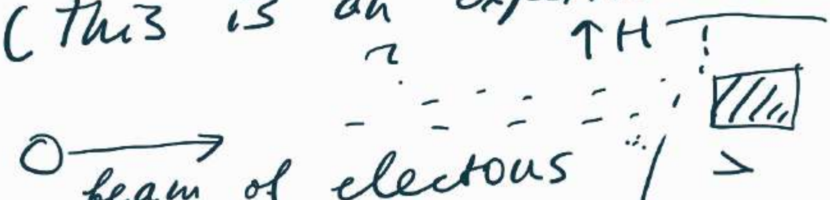
---

what happens when we measure observables like T_3 or T_1.

when we measure T_3 nothing happens with the state → we can continue measurements.

What happens when we measure T_1? and measurement give value say $+1$? We may measure again and sure we will get the same value.

(this is an experimental fact)



---

what happens with the state under

measurement?

Copenhagen interpretation saying that:
In the process of measurment
state probabilistic goes into the eigenvalue
of the observable:

Accord. to this interpretation, there are
two different processes

Process 1. Evolution: $|\psi\rangle \to e^{itH}|\psi\rangle$

unitary rotation, and it is
deterministic, no probabilities
given a state at $t=0$ we
definitely know state $t>0$

Process 2.: Strange process of measurement →

→ probabilistic $|\psi\rangle$ → goes to one of eigenstates $|i\rangle$ with probability w_i

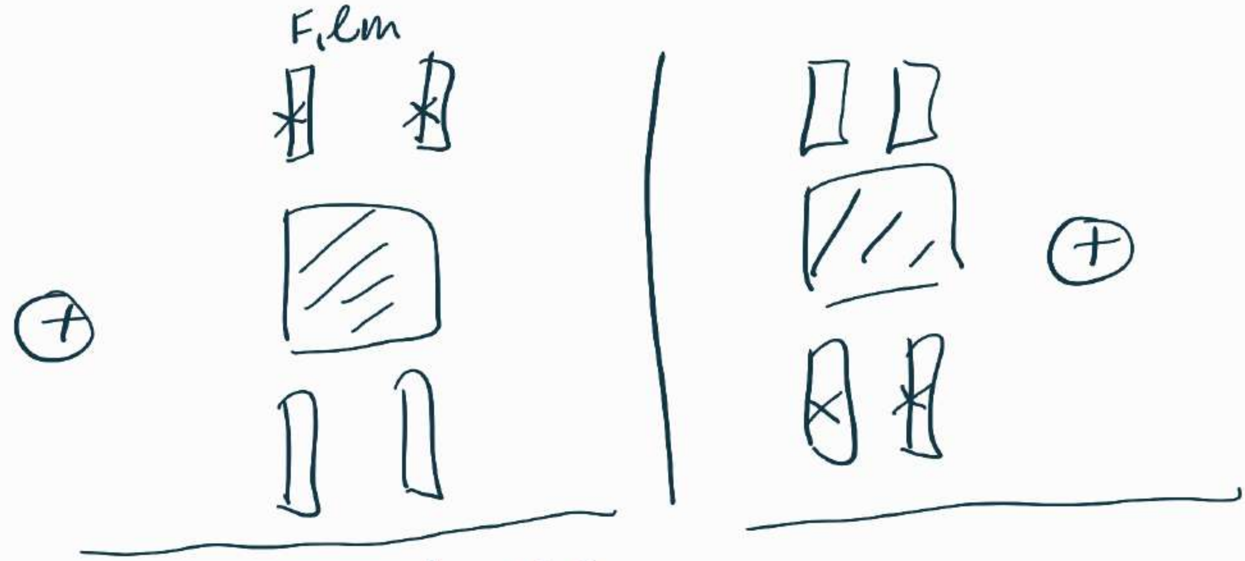
Question is – how this could happen?
Or. what is the difference between
evolution and measurement.

Maybe (as people in Copenhagen th.)
observable corresponds to a device that
is big while quantum system is
small?

Not true, since we observe big quantum effects in lasers and superconductors.

what happens?

Schrodinger cat puzzle after the break



Never have

single electron

Here we have it.

we cannot write class. trajec

no class trajectory

physical reality was divided into Q system and classical device.

But device should also be considered as quantum.

Extra concept

System 1 and System 2.

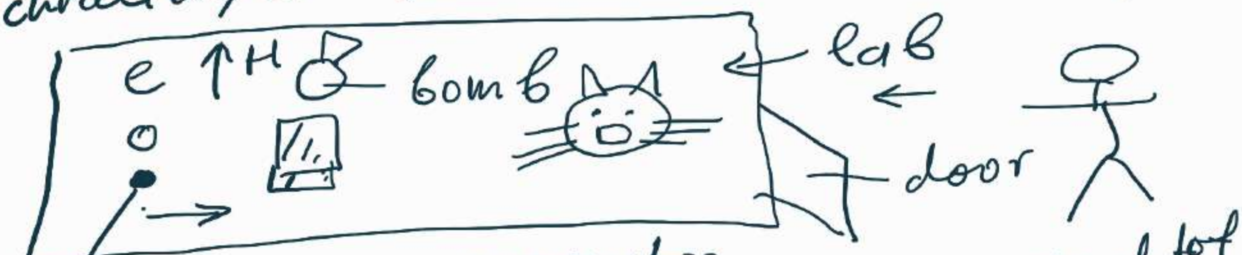
$\mathbb{C}P^{N_1-1}$ $\quad$ $\mathbb{C}P^{N_2-1}$.

System 1 and system 2 together, their space of states is

$$\mathbb{C}P^{N_1+N_2-1}.$$

By definition, system 1 and system 2 is also a physical system.

Schrodinger formulated a S. Cat puzzle



$\begin{pmatrix}1\\1\end{pmatrix} \otimes |start\rangle$ in magnetic field.

starting position

result of unitary rotation

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes |up\rangle + \begin{pmatrix}0\\1\end{pmatrix} \otimes |down\rangle$$

by $|\ \rangle$ I mean position of the c. of. m of electron

$$\left[\begin{pmatrix}1\\0\end{pmatrix} \otimes |up\rangle + \begin{pmatrix}0\\1\end{pmatrix} \otimes |down\rangle\right] \otimes |quite\ bomb\rangle \otimes |cat\rangle.$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes |up\rangle \otimes |exploded\ bomb\rangle \otimes \underline{|no\ cat\rangle}$$

$$+\begin{pmatrix}0\\1\end{pmatrix} \otimes |down\rangle \otimes |quite\ bomb\rangle \otimes |cat\rangle$$

System in the lab is a kind of superposition of a $|cat\rangle$ and a $|dead\ cat\rangle$.

How this could be is a puzzle

Humans could not imagine such a state

John von Newman who put it further (p.-ph. parallelism) & it was developed later by Everett,

$$\left[\begin{pmatrix}1\\0\end{pmatrix} \otimes |up\rangle + \begin{pmatrix}0\\1\end{pmatrix} \otimes |down\rangle\right] \otimes |quitt\ bomb\rangle \otimes |cat\rangle \otimes |$$  $\rangle$

$|world\rangle$

$=$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes |up\rangle \otimes |exploded\ bomb\rangle \otimes \underline{|no\ cat\rangle} \otimes |$$  $\rangle$

$$+\begin{pmatrix}0\\1\end{pmatrix} \otimes |down\rangle \otimes |quite\ bomb\rangle \otimes |cat\rangle \otimes |$$  $\rangle$

According to a unitary rotation $|world\rangle$ turns into a sum of two vectors: sad vector and happy vector

If another phys. comes to see what is going on we would get: ← world is sad, cat is dead

$|\ldots\rangle \otimes |☹\rangle \otimes |☹\rangle +$

$+ |\ldots\rangle \otimes |☺\rangle \otimes |☺\rangle$ ← world is happy, cat is alive!

multiworld interpretation of Quantum mechanics.

It says that probability comes from H. Beings. It comes from our perceptiom of the world.

This is an important thing for our life!

Really, $|😐\rangle$ have not one but many futures in the world.

All possibilities realize in the physical world.

$$|😐\rangle \xrightarrow{?} \begin{array}{c} |☺\rangle \\ \text{or} \\ |☹\rangle \end{array}$$

1) H. feings cannot die in their own perception. – there is always a possiblility of survival

It would be a miracle for others but the only way for them

2) Suicide is meaningless
(cons. of 1))

3) Question – are we responcible for all futures that we created?
Note, that futures do not intersect and do not interfere with each other.

whould be  that created this crazy experiment guilty for the death of the cat in future if he would never be there?

Quantum moral that is different from classical moral.
(Only because we say that the world is ruled only by unitary rotations)

Origine of probability is that <u>we</u> as h. beings have different futures in deterministic world.

(Relativity of probability)!

1959 Everette

$\alpha\, |\text{sad system}\rangle \otimes |☹\rangle +$ $\qquad \alpha \sim \frac{1}{10}$

$+\beta\, |\text{happy system}\rangle \otimes |☺\rangle$ $\qquad \beta \sim 1$

From the point of view of ☺ system can be only in the eigenstate ($|\text{sad}\rangle$ or $|\text{happy}\rangle$)

The norm of sad component is the probability that our future would be sad.