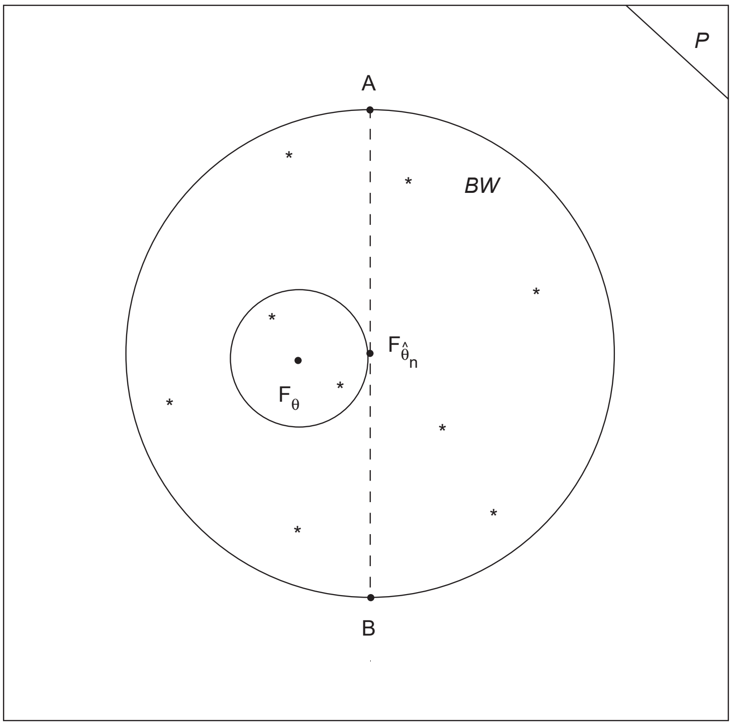


Fig. 3. $F_\theta$: true distribution, $F_{\hat{\theta}_n}$: smooth empirical distribution *: smooth boostrap empirical distributions. $BW$: the Bootstrap World, $P$: the World of Measures.

$\hat{\zeta}_n = E(\hat{\zeta}^*_{n,B}|\mathscr{X}_d)$ is the parameter to be estimated by $\hat{\zeta}^*_{n,B}$. The heuristics in Section 4.1 indicate that, as the dimension of the estimand $\zeta$ increases, the probability $F_{\hat{\zeta}^*_{n,B}}$ will stay in the Hellinger ball with centre $F_\zeta$ and radius $H(F_\zeta, F_{\hat{\zeta}_n})$ decreases. In Proposition 1, it is seen that the same holds for a larger radius. The probability is maximised when $F_\zeta$ is on the surface of the larger ball representing the Bootstrap World. That is, the bootstrap estimate of $F_\zeta$ (or of $\zeta$) has higher chance to beat the classical estimate when the latter deteriorates; as with the bootstrap samples, we will not know about it. In Example 1, the comparison of the differences $Dk$, $k = 1, 2$, was motivated by these heuristics.

## 5. Confirming the heuristics and the examples

**Proposition 1.** *Let* $\hat{\zeta}^*_{n,B,i}$ *be a bootstrap estimate of* $\zeta_i(\in R)$, *and let* $\hat{\zeta}_{n,i} = E[\hat{\zeta}^*_{n,B,i}|\mathscr{X}_d], =1,\ldots,d$; $\zeta, \hat{\zeta}_n, \hat{\zeta}^*_{n,B}$ *are the corresponding* $d$-*vectors. Then,*

(a) *the estimate* $\hat{\zeta}^*_{n,B}$ *is inadmissible*:

$$E\|\hat{\zeta}^*_{n,B} - \zeta\|^2 = E\|\hat{\zeta}_n - \zeta\|^2 + \sum_{i=1}^{d} E\,\mathrm{Var}(\hat{\zeta}^*_{n,B,i}|\mathscr{X}_d).$$