Fukaya category with stops

Let Λ be a conical subset of T^*M, that is one preserved by the scalar action on fibres. Then the above theorems raise the question: what is the image of the equivalence shown under the given restriction?

$$
\begin{array}{ccc}
D(\mathcal{Q}Con(M)) & \xrightarrow{\sim} & DFuk^{wr}(T^*M) \\
\cup & & \cup \\
D(\mathcal{Q}Con_\Lambda(M)) & \xrightarrow{\sim} & ?
\end{array}
$$

To answer this, we define the category "with stops" $DFuk^{wr}_{\Lambda}$, with objects given by Lagrangians which avoid Λ in the limit $|\xi| \to \infty$ for $(m, \xi) \in T^*M$.

Using this, we have the following recent result of Ganatra-Pardon-Shende [GPS]:

<u>Thm</u> $D(QCon_{\Lambda}(M)) \xrightarrow{\sim} DFuk^{wr}_{\Lambda}(T^*M)$

Toric homological MS

Combining the work of Kuwagaki and GPS, we obtain the following statement of homological MS:

$$
\begin{aligned}
& D(\mathrm{QCoh}(X_\Sigma)) && \text{(B-model)} \\
\xrightarrow{\sim}\ & D(\mathrm{QCon}_{\Lambda_\Sigma}(\mathbb{T})) && \\
\xrightarrow{\sim}\ & D\mathrm{Fuk}^{\mathrm{wr}}_{\Lambda_\Sigma}(T^*\mathbb{T}) && \text{(A-model)}
\end{aligned}
$$

Rem In fact, this work extends to toric stacks, in sense of, for instance, Geraschenko-Satriano [GS].

<u>Note</u> For 3-fold example we can take a resolution of $(xy - zw = 0) \subset \mathbb{C}^4$. See Donovan-Kuwagaki [DW], Section 4.2.

<u>Further topics</u>

See [T2] for correspondence between Dehn twist operations on symplectic manifolds and symmetries of $D(QCoh(X))$ under mirror symmetry.