Bulk Reconstruction in the Entanglement Wedge

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[Almheiri, XD, Harlow, JHEP 1504, 163 (2015)]

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### Anti-de Sitter/Conformal Field Theory Correspondence

- Best-understood model of quantum gravity
- Concrete example of emergent spacetime/gravity
- Easy to extract CFT quantities from the bulk
- Difficult to extract bulk quantities from the CFT
- Understand black hole interior?

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**Quantum gravity in AdS
d+1 (bulk)** | **Holographic CFTs on ∂AdS
d+1 (boundary)**
---|---
Isometry group $O(d, 2)$ | Conformal group $O(d, 2)$
Black hole states | Thermal states
Gauge symmetry | Global symmetry
States and operators | States and operators

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[Anti-de Sitter/Conformal Field Theory Correspondence](Maldacena '97)
What operator in CFT represents a local bulk operator?
Once we know this, we have full access to bulk information.
Answering this question helps us reconstruct the bulk.
Global AdS reconstruction

\[ \phi(x) = \int_{S^{d-1} \times \mathbb{R}} dY K(x; Y) \mathcal{O}(Y) \]

[Hamilton, Kabat, Lifschytz & Lowe ’06]

• \( O(1/N) \) corrections
Global AdS reconstruction

\[ \phi(x) = \int_{S^{d-1} \times \mathbb{R}} dY K(x; Y) O(Y) \]

[Hamilton, Kabat, Lifschytz & Lowe ’06]

• \(O(1/N)\) corrections

• Reconstruct bulk operators from a limited set of CFT data?

“Subregion duality”
AdS-Rindler reconstruction for disk $A$

$$\phi(x) \sim \int_{D[A]} dY K_A(x; Y) \mathcal{O}(Y)$$
What region of the dual spacetime is described by a general subregion in a holographic CFT?

- HKLL works only in (smaller) causal wedge.
What region of the dual spacetime is described by a general subregion in a holographic CFT?

- HKLL works only in (smaller) causal wedge.
- Conjecture: bulk reconstruction works in (larger) entanglement wedge.
- Goes beyond black hole horizon and reconstructs interior.
Holographic Entanglement Entropy

A simple and powerful prescription for entanglement entropy:

\[ S = \frac{\text{Area}(\text{Minimal Surface})}{4G_N} \]

Recall the definition:

\[ S \equiv -\text{Tr}(\rho_A \ln \rho_A) \]

Covariant generalization:

[Hubeny, Rangamani & Takayanagi ’07]  [XD, Lewkowycz & Rangamani 1607.07506]
Reconstruction conjecture for entanglement wedge

- **Entanglement wedge** is a bulk region bounded by the Ryu-Takayanagi minimal surface.
- It may change discontinuously.
- Conjecture: Any bulk operator in entanglement wedge of $A$ may be represented as a CFT operator on $A$.

[Czech, Karczmarek, Nogueira & Van Raamsdonk ’12]  [Wall ’12]
[Headrick, Hubeny, Lawrence & Rangamani ’14]
Conjecture:
Any bulk operator in entanglement wedge of $A$ may be represented as a CFT operator on $A$.

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<th>Proving this conjecture</th>
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<td>Quantum error correction</td>
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CFT relative entropy = bulk relative entropy

Bulk operator in EW commutes with any $X_{\overline{A}}$

Reconstruction in entanglement wedge
Why quantum error correction?

• $\phi(x)$ can be represented on $A \cup B$, $B \cup C$, or $A \cup C$.
• Obviously they cannot be the same CFT operator.

[Almheiri, XD, Harlow ’14]
Why quantum error correction?

- $\phi(x)$ can be represented on $A \cup B$, $B \cup C$, or $A \cup C$.
- Obviously they cannot be the same CFT operator.
- Defining feature for quantum error correction.
- Holography is a quantum error correcting code.
- Reconstruction works in a code subspace of states.

[Almheiri, XD, Harlow ’14]
Three-qubit model

• Alice wants to send a qudit by mail.
• She encodes it into the Hilbert space of 3 qudits.

$$\tilde{0} = \frac{1}{\sqrt{3}} (|000\rangle + |111\rangle + |222\rangle)$$

$$\tilde{1} = \frac{1}{\sqrt{3}} (|012\rangle + |120\rangle + |201\rangle)$$

$$\tilde{2} = \frac{1}{\sqrt{3}} (|021\rangle + |102\rangle + |210\rangle)$$

• These states span the code subspace.
• In holography, code subspace contains bulk states.

[Almheiri, XD, Harlow '14]
Ryu-Takayanagi with bulk quantum corrections

\[ S = \frac{\text{Area(Minimal Surface)}}{4G_N} + S_{\text{bulk}} \]

- \( S_{\text{bulk}} \): von Neumann entropy of entanglement wedge
- Also has higher derivative corrections. \([\text{XD '13}] \ldots\]
- Derived by FLM up to \( O(G_N) \) corrections; conjectured true to all orders. \([\text{Engelhardt & Wall '14}] \quad [\text{XD & Lewkowycz, to appear}]\)
- Intuitively, \( \rho_A \) has information in entanglement wedge of A!
CFT relative entropy is bulk relative entropy

- Define a subspace $H_c$ of states where bulk EFT is valid.
- E.g. $H_c = \{\text{All states with } E < M_{Pl} \text{ (& conformal images)}\}$.
- Will call it a code subspace.
- Rewrite RT with quantum corrections:

\[
S(\rho_A) = \text{Tr}(\rho_a A_{\text{loc}}) + S(\rho_a)
\]

- Small change of state $\rho \rightarrow \rho + \delta \sigma$ in $H_c$:

\[
\text{Tr}(\delta \sigma_A K_{\rho_A}) = \text{Tr}[\delta \sigma_a (A_{\text{loc}} + K_{\rho_a})]
\]

- Modular Hamiltonian: $K_\rho \overset{\text{def}}{=} - \log \rho$.

- Integrate $\delta \sigma$:

\[
\text{Tr}(\sigma_A K_{\rho_A}) = \text{Tr}[\sigma_a (A_{\text{loc}} + K_{\rho_a})]
\]
CFT relative entropy is bulk relative entropy

- From
\[ \text{Tr}(\sigma_A K_{\rho_A}) = \text{Tr}[\sigma_a (A_{\text{loc}} + K_{\rho_a})] \]

- Relative entropy: \( S(\rho|\sigma) \overset{\text{def}}{=} \text{Tr}(\rho \log \rho) - \text{Tr}(\rho \log \sigma) \)

- States are as distinguishable in the bulk as in the CFT.
- Intuitively, this means we must be able to reconstruct in entanglement wedge.

\[ S(\rho_A|\sigma_A) = S(\rho_a|\sigma_a) \]

[Jafferis, Lewkowycz, Maldacena & Suh 1512.06431]
Conjecture:
Any bulk operator in entanglement wedge of $A$ may be represented as a CFT operator on $A$.

Proving this conjecture

| Quantum error correction | RT with bulk quantum corrections |

CFT relative entropy = bulk relative entropy

Bulk operator in EW commutes with any $X_{\bar{A}}$

Reconstruction in entanglement wedge
A reconstruction theorem

• Goal is to prove: \[ \langle \phi | [O_a, X_A] | \phi \rangle = 0 \]

• This is necessary and sufficient for

\[ \exists O_A, \text{s.t. } O_A |\phi\rangle = O_a |\phi\rangle \text{ and } O_A^\dagger |\phi\rangle = O_a^\dagger |\phi\rangle \]

• WLG assume \( O_a \) is Hermitian.

• Consider two states \(|\phi\rangle, e^{i\lambda O_a} |\phi\rangle\) in \( H_c \):

\[ S(\rho_A |\sigma_A) = S(\rho_{\bar{a}} |\sigma_{\bar{a}}) = 0 \]

\[ \langle \phi | e^{-i\lambda O_a} X_A e^{i\lambda O_a} |\phi\rangle = 0 \]

[Almheiri, XD, Harlow ’14]

[XD, Harlow & Wall 1601.05416]
Explicit reconstruction (in principle)

• Add a reference system $R$ to CFT

$$|\Psi\rangle = \sum_i |i\rangle_R \otimes |i\rangle_{A\bar{A}}$$

• Can mirror $O_a$ to an operator $O_R$.

• View $O_R$ as $O_R \otimes I_{\bar{A}}$ and mirror it back onto $A$ as $O_A$.

• Use Schmidt decomposition: $|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\alpha\rangle_A \otimes |\alpha\rangle_{R\bar{A}}$.

• Obstruction: $O_R \otimes I_{\bar{A}}$ may mix zero $c_{\alpha}$ with nonzero ones.

• This cannot happen if

$$[O_R \otimes I_{\bar{A}}, \rho_{R\bar{A}}] = 0$$

$$\langle \phi | [O_a, X_{\bar{A}}] |\phi\rangle = 0$$

[Almheiri, XD, Harlow ’14]
Theorem: Any bulk operator in entanglement wedge of $A$ may be represented as a CFT operator on $A$.

Proving this conjecture

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Reconstruction in entanglement wedge
What We Learned

• Quantum information theory enables us to understand the basic dictionary of quantum gravity.

• Viewing holography as a quantum error correcting code, we can analyze how to “build spacetime from entanglement”.
Future Directions

• **Simple** explicit reconstruction of bulk operators in the entanglement wedge?

• How do we enjoy all of this?

✓ Study **black hole interior** and information paradox?

✓ Understand better the emergence of spacetime and gravity?