DFT
A Pathway to Quantum Strings

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I am NOT going to talk about higher-spin theory.

- colored higher-spin gauge theory
- partially massless Higgs mechanism

Rather, I would like to share with you thoughts on something ”explorative” and ”wild”.
▶ ArXiv 1507.07545
▶ ArXiv 1510.06735
▶ Work in progress 1608.nnnnn
▶ In collaboration with
  Kanghoon Lee, Jeong-Hyuck Park, Woohyun Rim (Seoul)
  Yuho Sakatani (Kyoto), Alejandro Rosabal (Buenos Aires)

cf. talks by Arkani-Hamed, Yu-Tin Huang
A Student Project (I): Calculable & Predictable?

\[ I = \int d^4 x \left[ \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} (\partial \psi)^2 + \frac{1}{2} g^2 \phi^4 (\partial \phi)^2 + \frac{1}{2} \lambda^2 \psi^4 (\partial \psi)^2 \right. \\
+ \frac{1}{2} (\ln 2)^2 \kappa^2 \left( \frac{1}{7} \phi^4 \psi^7 (\partial \phi) + \frac{1}{5} \phi^5 \psi^6 (\partial \psi) \right)^2 + \frac{1}{2} \zeta^2 (7) h^2 \psi^{16} (\partial \psi)^2 \\
+ \frac{1}{2} (\ln 3)^2 \eta^2 \left( \frac{1}{9} \phi^9 \psi^4 (\partial \psi) + \frac{1}{5} \phi^8 \psi^5 (\partial \phi) \right)^2 + \frac{1}{2} \zeta^2 (4) \sigma^2 \phi^{14} (\partial \phi)^2 \\
+ \frac{1}{3} g (\partial \phi) (\partial \phi^3) + \frac{1}{2} \lambda (\partial \psi) (\partial \psi^2) + [\zeta (7) h \psi^8 + \zeta (4) \sigma \phi^7] (\partial \phi \partial \psi) \\
+ \frac{\ln 2}{35} \kappa (\partial \phi) \partial (\phi^5 \psi^7) + \frac{\zeta (7)}{27} gh (\partial (\phi^3)) \partial (\psi^9) + \frac{\ln 2}{105} g \kappa \partial (\phi^3) \partial (\phi^5 \psi^7) \\
+ \frac{\ln 3}{45} \eta (\partial \psi) \partial (\phi^9 \psi^5) + \frac{\zeta (4)}{16} \lambda \sigma \partial (\psi^2) \partial (\phi^8) + \frac{\ln 3}{90} \lambda \eta \partial (\psi^2) \partial (\psi^5 \psi^9) \\
+ \frac{\ln 2 \zeta (7)}{315} h \kappa \partial (\phi^9) \partial (\phi^5 \psi^7) + \frac{\ln 3 \zeta (4)}{360} \sigma \eta \partial (\phi^8) \partial (\phi^9 \psi^5) \\
+ \text{(add extra if needed)} \]
Secret code to Einstein gravity

\begin{align*}
I_{\text{gravity}} & = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R(g) + \text{(Gibbons-Hawking)} \\
& = \int d^4x \left[ (\partial h)^2 + h^2 (\partial h)^2 + h^3 (\partial h)^2 + \cdots \right]
\end{align*}

In particular,

Divergence at $\ell$-loop $= \ell(d - 2) + 2$,

independent of the number of external lines.
After canonical transformation

\[ \Phi = \phi + \frac{g}{3} \phi^3 + \frac{\zeta(7)}{9} h \psi^9 + \frac{\ln 2}{35} \kappa \phi^5 \psi^7 \]

\[ \Psi = \psi + \frac{\lambda}{2} \psi^2 + \frac{\zeta(4)}{8} \sigma \phi^8 + \frac{\ln 3}{45} \eta \phi^9 \psi^5, \]

the theory is **FREE**: 

\[ I_{\text{student}} = \frac{1}{2} (\partial \Phi)^2 + \frac{1}{2} (\partial \Psi)^2 \]

Ward identities:

\[ \frac{Z_{\phi^3}}{Z_{\phi}} = \frac{Z_{\psi^9}}{Z_{\phi^3}} = \frac{Z_{\phi^5 \psi^7}}{Z_{\phi^3} Z_{\psi^9}} = 1, \quad \frac{Z_{\psi^2}}{Z_{\psi}} = \frac{Z_{\phi^8}}{Z_{\psi^2}} = \frac{Z_{\phi^9 \psi^5}}{Z_{\phi^8}} = 1 \]
Student Project (II): Calculable and Predictable?

O(N) nonlinear sigma model:

\[ I = \int d^4x \sum_{a,b=1}^{N-1} G_{ab}(\phi) \partial \phi^a \partial \phi^b, \quad G_{ab} = \delta_{ab} + \frac{\phi_a \phi_b}{F^2 - \phi^2} \]

The model has two-derivative, non-polynomial interactions. At tree level, it behaves badly at real energy scales above \( \sim 4\pi F \). At loop level, it behaves badly at virtuality above \( \sim 4\pi F \).

To get better UV behavior, one must append the model with additional degrees of freedom at the threshold scale \( \sim 4\pi F \). This UV completion is done by adding a Higgs field \( \sigma \) as the \( N \)-th component, replacing the scale \( 4\pi F \):

\[ I = \int d^4x \sum_{A=1}^N \partial \Phi_A \partial \Phi_A + \frac{\lambda}{4 \sum_{A=1}^N (\Phi_A \Phi_A - F^2 / \lambda)^2} \]
Maxim from Student Project

To attain better high-energy behavior and renormalizability, calculability and hence predictability of a theory framework:

- Enlarge field degrees of freedom
- Choose Variables Smartly
- Enlarge gauge/global symmetries
Quantum Theory of Gravity

- String theory constructed as a theory of quantum gravity
- Prohibitively complicated, so study by "divide & conquer"
- Couplings, kinematical asymptotics, gauge symmetries

Starting from Einstein gravity, take pathway to string theory via:

1. \((g, b, \phi)\) supergravity = infinite tension, low-energy limit
2. \((g_2, g_3, \cdots)\) higher-spin = zero tension, high-energy limit
3. Double field theory = enlarged gauge symmetry limit
Remark on Massive Higher Spins and Strings

- Causality constraint $\rightarrow$ a tower of massive higher spins
  Maldacena et.al.

- Is the tower of massive higher spins to be automatically equated to string theory?

Consider $\text{AdS}_{d+1}$ Vasiliev theory compactified to $\text{AdS}_d \otimes$ Janus geometry with two boundaries Bak, Gutperle, Hirano

- infinite Kaluza-Klein tower of massive higher spins in $\text{AdS}_d$
  Gwak, Kim, SJR

- Shapiro time delay in $\text{AdS}_d$ necessitates higher spins

- At UV, they come from $\text{AdS}_{d+1}$ Vasiliev, not from string (yet)
Roadmap for Higgs Hunt

String Theory

A Pathway?

Einstein Gravity
Pathway through Double Field Theory (DFT)

Einstein gravity \subset Double Field Theory \subset String Theory

\[ Diff(M_D) \subset Diff(M_{2D}) \subset G_{\text{string}} \]

[Question]

How (much) is high-energy behavior improved?

[Note] DFT as a proxy for the MAXIM
DFT - Spacetime Description

- start with T-duality symmetry of strings
- treat momentum, winding on equal footing: \((x^m, \tilde{x}_m) := X^M\)

\[
\mathcal{M}_D \otimes \tilde{\mathcal{M}}_D \rightarrow \hat{\mathcal{M}}_{(D,D)} \rightarrow \overline{\mathcal{M}}_D / O(D, D, \mathbb{Z})
\]

\[
\partial_A \partial^A \Phi(X) \simeq 0, \quad \partial_A \Phi(X) \partial^A \psi(X) \simeq 0.
\]

- \((g, b, \phi)_D \rightarrow (g, b, \phi)_{(D,D)}, \) then project down to \(\overline{\mathcal{M}}_D\)
- combine \(\text{Diff}(g)\) and \(\text{G}(b)\) with \(O(D,D)\) covariance

\[
G_{\text{DFT}} = \text{Diff}(\hat{\mathcal{M}}_{(D,D)}); \quad \text{Diff}(\hat{\mathcal{M}}_{(D,D)}) \gg \text{Diff}(\mathcal{M}_D)
\]

- DFT endows enormously enlarged gauge invariance to the massless string modes \((g, b, \phi)\), at the apparent expense of manifest locality
DFT – Worldsheet Description

- \( x(z, \bar{z}) = x_L(z) + x_R(\bar{z}); \)  \( \tilde{x}(z, \bar{z}) = x_L(z) - x_R(\bar{z}) \)
- T-duality

\[
\begin{align*}
    x_L & \rightarrow x_L, & x_R & \rightarrow -x_R
\end{align*}
\]

is the origin of \( O(D, D) \) signature of \( X^M = (x, \tilde{x}) \)

- Level-matching condition leads to the section constraints

\[
(L_0 - \bar{L}_0)\Phi = 0 \quad \Rightarrow \quad \partial^2 \Phi = 0, \quad \partial_A \Phi \partial^A \Psi = 0
\]

- \( G_{\text{DFT}} = \text{Diff}(\hat{\mathcal{M}}_{(D,D)}) \) arises from closed SFT gauge algebra, which receives \( \alpha' \)-corrections after \( O(D, D) \) non-covariant field and parameter redefinitions
DFT at Leading Order in $\alpha'$

- Fields $(g, b, \phi) \rightarrow (\mathcal{M}_{MN} = \mathcal{E}_M \cdot \mathcal{E}_N, d)$
- Smart choice of variables
- O(D,D) invariant metric and O(D,D) covariant background

\[ \mathcal{J} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \mathcal{H} = \langle \mathcal{M} \rangle \]

- At leading-order in $\alpha'$, $\mathcal{M}$ is constrained
  - $\mathcal{M}^2 = \mathcal{J}$
  - $\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{pmatrix}$

\[ S_0 = \int e^{-2d} R(\mathcal{E}) + \oint e^{-2d} L_{GH}(\mathcal{E}) \quad \text{subject to} \quad \mathcal{M}^2 = \mathcal{J} \]

\[ \mathcal{R} := \mathcal{M}\partial\mathcal{M}\partial\mathcal{M} + \partial\partial\mathcal{M} + \mathcal{M}\partial d\partial d + \partial\mathcal{M}\partial d + \partial\mathcal{E}\partial\mathcal{E}\mathcal{M} \]

- The action is uniquely fixed by $G_{DFT}$!
- Enlarged gauge symmetry

- Unfortunately, the constraint $\mathcal{M}^2 = \mathcal{J}$, puts the weak field expansion of $\mathcal{M}$ as $g = \eta + h$ and $g^{-1} = \eta + h + h^2 + \cdots$ non-polynomial.
At leading-order in $\alpha'$, the DFT achieves enormously enlarged gauge symmetry $G_{\text{DFT}}$, but the field variables are still not smart

DFT at leading order in $\alpha' \simeq$ Einstein gravity
DFT at Next Order in $\alpha'$

- Metaphor: Nonlinear to Linear O(N) model with Higgs
- At next order, DFT miraculously manages to achieve this:

\[
S_1 = \int e^{-2D\left[\left(\mathcal{M} - \frac{1}{3}\mathcal{M}^3\right) + \alpha'\left((\mathcal{M}^2 - 1)\mathcal{M}\partial\partial\mathcal{D} + \mathcal{M}\partial\mathcal{M}\partial\mathcal{M} + \mathcal{M}\partial\partial\mathcal{D}\right) + O(\alpha'^2 \partial^4) + O(\alpha'^3 \partial^6)\right]}
\]

- Enlarged gauge symmetry remains the same, $G_{\text{DFT}}$
- $\mathcal{M}$ is no longer constrained; $S_1$ is polynomial in fields
- The action is uniquely determined by $G_{\text{DFT}}$
- No other possible counter-terms up to this order

- What are the "Higgs" modes?
- Is high-energy behavior improved?
Higgs modes?

- $\mathcal{M}$ is no longer constrained
- $S_0$ contains $\partial^2$ terms, $S_1$ contains up to $\partial^6$ terms.
- $\mathcal{M} \approx (g, b, \phi) \oplus (m, \bar{m})_{(mn)}$ from weak field expansion
- By O(D,D) field redefinition, only $\partial^2$ terms are relevant

\[ \alpha' \Box m - 4m = \mathcal{L}(\mathcal{M}, m, \bar{m}) = h^T h + \cdots \]
\[ \alpha' \Box \bar{m} + 4\bar{m} = \bar{\mathcal{L}}(\mathcal{M}, m, \bar{m}) = h h^T + \cdots \]

- It can be viewed as quiver matrix theory associated with double spin-Lorentz symmetry O(D)$\otimes$O(D). [Arkani-Hamed+Kaplan]
  The $m, \bar{m}$ fields are symmetric, the $(h, b)$ is bi-fundamental. It admits large-$D$ expansion. [cf. Strominger, Emparan, Minwalla, higher-spin theory]
Higgs Modes?

Extra fields \((m, \bar{m})\) are the "Higgs" fields with features:

- \(\mathcal{M}\) is no longer constrained; extra DOFs = \(m, \bar{m}\)
- \(m_0i, \bar{m}_0i\) are non-dynamical fields \(\text{cf. not Lagrange multiplier}\)
- negative norm, akin to Pauli-Villar and Lee-Wick
- \(m^2 = \pm 4/\alpha';\) this precise massive pair is needed upon integrating out \(m, \bar{m}\) to cancel \(\partial^0\) terms for \(\mathcal{M}\) absent in \(S_0\)

\(\text{cf. Hohm, Nasser, Zwiebach}\)
High-Energy Behavior (I)

- 4-point \((h, b)\) amplitudes

\[
A_4^{\text{DFT}}(s, t, u) \sim \left(1 + \frac{su}{s^2 - 4/\alpha'^2} + \cdots \right) A_4^{\text{grav}}(s, t, u)
\]

- KLT kernel

\[
A_4^{\text{DFT}} = A_4^{\text{chiral}}(+\eta) K_{\text{DFT}} \tilde{A}_4^{\text{chiral}}(-\eta)
\]

where

\[
K_{\text{DFT}} = \left(1 + \frac{su}{s^2 - 4/\alpha'^2} + \cdots \right) K_{\text{grav}}
\]

- For such soft UV behavior, both "negative-norms" and "precise mass-squared spectrum \(\alpha' m^2 = \pm 4\) are crucial
High Energy Behavior (II)

- BCFW factorization viewed as soft-collinear scattering
  Arkani-Hamed+Kaplan

- At large $z$, enhanced spin symmetry governs the leading behavior

- For leading DFT, the same as Einstein gravity cf. Boels+Hurst

- For $O(\alpha')$ DFT, BCFT asymptotics gets more convergent

$$A_4^{\text{DFT}}(-, -; \pm, \pm) \to \left( \frac{1}{Z} + \cdots \right) \cdot \frac{1}{Z^s} \bigg|_{s=2}$$

- Similar softer behavior for other polarizations
High Energy Behavior (III)

- vacuum amplitudes

\[ A_0 = \int d^4 p \left[ \sum_{(h,b,d)} \log p^2 - \sum_{m,\bar{m}} \log(p^2 \pm 4) \right] \]

\[ \simeq \left[ (d - 2)^2 - 2 \cdot \frac{1}{2} (d - 1)^2 + 2(d - 1) + 1 \right] \Lambda^4 + \cdots \]

- Subleading divergence uncancelled
- Radiative correction to Newton’s constant non-vanishing from leading order

cf. bosonic YM theory at d=26; Tseytlin, SJR
At higher-order in $\alpha'$, the DFT retained enormously enlarged gauge symmetry $G_{\text{DFT}}$, and also the "Higgs" modes that soften the high-energy behavior.

DFT at higher order in $\alpha' \simeq$ UV improved gravity

cf. Higher-spin theory at higher loop order

Giombi, Klebanov, Tseytlin, Beccaria, ....
Remark on Indefinite Hilbert Space

- $m, \bar{m}$ are ghosts
- dynamical Pauli-Villar fields and Lee-Wick mechanism?
  - Boulware, Gross; Grinstein, Wise
- If arising from a certain limit of string theory, then ”how”?
- Could they be viewed as ”effective” description of contribution of infinitely many positive-norm states?

\[
2 \sum_{s=1}^{\infty} \frac{+1}{p^2 \pm m^2} = \frac{2\zeta(0)}{p^2 \pm m^2} = \frac{-1}{p^2 \pm m^2}
\]
Remarks for Ambitwistor Strings

- $O(\alpha')$ DFT was derived from chiral CFT and hence ”chiral string” dynamics \cite{HohmSiegelZwiebach}

- We derived this chiral string from conventional string after integrating out anti-chiral part and taking infinite tension limit; This fits nicely on ”how” the spacetime signature changes between original and T-dual coordinates in DFT coordinates $X^M = (x, \tilde{x})$ \cite{Hai-TangYang}

\[
\langle x^m(z, \bar{z}) \tilde{x}^n(z, \bar{z}) \rangle = \eta^{mn} \log \frac{z}{\bar{z}} = +\eta^{mn} \log z - \eta^{mn} \log \bar{z}
\]

- The signature change converts the standard KLT to closed string amplitude to DFT amplitude \cite{HuangSiegelYuan}

- Ambitwistor string approach to YM and gravity scattering amplitudes and explanation of CHY scattering equation \cite{CachazoHeYuan,CasaliTourkine}
Remark on Little DFT

- LST enjoys T-duality symmetry \(^Vafa et.al.;\) Hohenegger, Iqbal, SJR; Kim, Kim, Lee
- no ten-dimensional \((g, b, \phi)\)
- 5-brane worldvolume fields \((a, b^+)\) + massive excitations
- manifest O(D,D) covariant description of \((a, b^+)\) leads to **doubled gauge theory** with enlarged gauge symmetries

\[
S_{\text{littleDFT}} = \int e^{-2d} \frac{1}{2} \text{Tr} F^2(A), \quad A_M \sim (a, b^+) 
\]

- As a pathway for UV completing gauge theories to LST, explore high-energy behavior of the little DFT
- Expect to shed light to noncritical / QCD strings \(^\text{cf. Komargodski}\)
- Tension between massive higher-spins from QCD \((a)\) versus from abelian Higgs model \((b^+)\)?
Thank You

For out of olde feldes, aas men seith,
Cometh al this newe corn fro yeer to yere;
And out of olde bokes, in good feith,
Cometh al this newe science that men lere.

Geoffrey Chaucer