Higher Spin de Sitter Holography

Frederik Denef

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work in collaboration with

Dio Anninos, Ruben Monten, Zimo Sun
Motivation

From the combination of the fundamental constants, $G$, $c$, and $h$ it is possible to form a new fundamental unit of length $L_{\min} = 7 \times 10^{-28} \text{cm}$. It seems to be inevitable that this length must play some role in any complete interpretation of gravitation. [...] In recent years great progress has been made in knowledge of the excessively minute; but until we can appreciate details of structure down to the quadrillionth or quintillionth of a centimetre, the most sublime of all the forces of Nature remains outside the purview of the theories of physics. (Eddington 1918, 36)

[Dean Rickles, Pourparlers for Amalgamation ...]

de Sitter is mysterious
We will not say much more about de Sitter space in this course. A big reason for this is that we don’t have any UV-complete theory of gravity in de Sitter, like we do in anti-de Sitter. We also have no clear answer to the question ‘What is the de Sitter entropy?’ (Does it count the microstates of something?) Since we live in de Sitter, this seems like a very important question.

[Tom Hartman, Lectures on Quantum Gravity, 2015]
Motivation

String Theory:
- No classical dS solutions [Maldacena-Nuñez, de Wit et al]
- Quantum dS solutions [Kachru-Kallosh-Linde-Trivedi]
- Landscape [Bousso-Polchinski, Susskind, Douglas]

The Great Wall:
- No classical dS = no semiclassical control [Dine-Seiberg]
- No complete theory = no predictions (e.g. [FD-Douglas '04]
distribution susy breaking scale: \(d\mathcal{N} \sim F^5 dF \Rightarrow ?\))
- dS-CFT? [Strominger, Witten, Maldacena]
  Examples? [Silverstein, Anninos-Hartman-Strominger]
  Consensus: troubling [Susskind-Kleban et al]
From AdS to dS:

- Continuation $L \to iL$, $z \to i\eta$:

  $$ds^2_{\text{AdS}} = \frac{L^2}{z^2} (dz^2 + dx^2) \quad \rightarrow \quad ds^2_{\text{dS}} = \frac{L^2}{\eta^2} (-d\eta^2 + dx^2).$$

- Boundary = future infinity $\eta = -\frac{1}{L} e^{-t/L} = 0$.

- $\lim_{\eta \to 0} \phi(\eta, x) \sim \eta^\Delta \alpha(x) + \eta^{d-\Delta} \beta(x)$
dS-CFT

Quantization:

- \( \lim_{\eta \to 0} \phi(\eta, x) \sim \eta^\Delta \alpha(x) + \eta^{d-\Delta} \beta(x) \)
- \([\hat{\alpha}(x), \hat{\beta}(y)] \sim i \delta^d(x - y)\).
- Dirichlet/Neumann future boundary states: \( \hat{\alpha}|D\rangle = 0, \hat{\beta}|N\rangle = 0 \).

**dS-CFT [Strominger]:**

\[
\langle D|\hat{\beta}_1 \cdots \hat{\beta}_n|0\rangle = \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_{\text{CFT}_D} , \quad \langle N|\hat{\alpha}_1 \cdots \hat{\alpha}_n|0\rangle = \langle \tilde{\mathcal{O}}_1 \cdots \tilde{\mathcal{O}}_n \rangle_{\text{CFT}_N}
\]

**dS-CFT 2.0 [Maldacena]:**

\[
\Psi_{\text{HH}_D}[\alpha] = \langle \alpha|0\rangle = \langle D|e^{i \int \alpha \hat{\beta}}|0\rangle = \langle e^{i \int \alpha \mathcal{O}} \rangle_{\text{CFT}_D} = Z_{\text{CFT}_D}[\alpha]
\]

Note \( \hat{\alpha}|\alpha\rangle = \alpha|\alpha\rangle \). Analogous \( \Psi_{\text{HH}_N}[\beta] = \) Fourier transform \( \Psi_{\text{HH}_D}[\alpha] \).
Challenges for dS-CFT

Unconventional, (very) nonunitary CFT:

- $\langle O O \rangle = \partial\partial \log \Psi_{HH}\big|_0 < 0 \Rightarrow$ CFT inner product not $> 0$.
- Central charge $C_{\text{AdS}_{d+1}} \sim \frac{L^{d-1}}{G} \rightarrow C_{\text{dS}_{d+1}} \sim i^{d-1} \frac{L^{d-1}}{G}$ not $> 0$.
- Scalar mass $m^2_{\text{AdS}} = \frac{\Delta(\Delta-d)}{L^2} \rightarrow m^2_{\text{dS}} = -\frac{\Delta(\Delta-d)}{L^2}$.

⇒ Usual abundance of large $\Delta$ operators like $\text{Tr} \ X^n$ would lead to abundance of tachyons + heavy particles have complex $\Delta$:

$\Delta = \frac{d}{2} \pm i \sqrt{(mL)^2 - \left(\frac{d}{2}\right)^2}$.

Note:

- Not inconsistent: Euclidean CFT.
- $\sim$ CFT analog no-go dS brane constructions.
- Examples? $\sim$ Bootstrap? What replaces unitarity constraint?
Objections raised by [(Dyson/Goheer)-Kleban-Susskind] point to trouble for dS-CFT as a complete theory:

- Boltzman brains.
- Eternal dS does not appear to exist in string theory. Connected to entire landscape, ultimately decays into zero or negative cc vacua.
- Tension between features of dS representation theory and finiteness of dS horizon entropy.
Challenges for dS-CFT

Bulk quantum mechanics?

- Time is emergent. Bulk Hilbert space $\neq$ CFT state space. Bulk Hamiltonian $\neq$ CFT Hamiltonian.

- Different CFTs (D/N) for $\hat{\beta}$ and $\hat{\alpha}$ correlators. What about mixed $\hat{\alpha}\hat{\beta}$ correlators? Unified in one QFT? **Commutator problem:**

$$\langle \hat{\alpha}(x) \hat{\beta}(y) \rangle = i \delta^d(x - y) = -\langle \hat{\beta}(y) \hat{\alpha}(x) \rangle$$

$\sim$ Different operator orderings in Euclidean QFT?? $\times$

- CFT supposedly gives “wave function” $\Psi[\phi] = \langle \phi | 0 \rangle$ but:
  - Computing $\langle 0 | \cdots | 0 \rangle$ correlators requires integrating over $\phi$.
  - Probabilities? Measure? Phase space? Wave function of what?
  - Bulk dynamics?

[ArkaniHamed-Maldacena]
Aside: Cosmic Clustering

Free massless scalar (or metric) in dS:

1. Effective stochastic time evolution: branched diffusion [Starobinsky].
2. Wave function $\Psi[\phi] \sim e^{-\int d^d k k^d \phi_k \phi_{-k}}$

(1) can be cleanly detected in (2) by computing a “phylogenetic” triple overlap distribution introduced in study of spin glasses [Anninos-FD]
Goal of this work

We want to find an **exact and complete** model for dS holography.

Interesting and useful things can be said at approximate level, e.g. by analytic continuation from AdS-CFT

[Skenderis-McFadden-Bzowski, Hartle-Hertog-Hawking, ...].

But we want complete, non-perturbative formulation. In exchange we will drop requirement of phenomenological realism.
AHS ghost models

- Higher spin $\text{AdS}_4 - U(N)$ vector model correspondence
  [Klebanov-Polyakov, Giombi-Yin] $\rightarrow$ higher spin $\text{dS}_4 - U(N)$ vector model correspondence [Anninos-Hartman-Strominger]. Instead of bosonic scalars, fermionic scalars.

- Free model in 3d:
  \[ S = \int d^3x \left| \partial V^A \right|^2 \quad (U(N) \text{ singlet sector}). \]

One single trace scalar primary $\mathcal{O} = :V^A\bar{V}^A: \quad \text{with } \Delta = 1.$

- Dual to Vasiliev $\text{dS}_4$ containing massless spin $s$ particles for all $s = 0, 1, 2, \ldots.$ Scalar of $m^2 = \frac{2}{L^2}$, Neumann future boundary state.

- $m^2 > 0 \quad \checkmark, \quad C = -N \quad \checkmark, \quad \langle \mathcal{O}\mathcal{O} \rangle < 0 \quad \checkmark$

- $\Rightarrow \Psi_{HH}[\beta, \ldots] = \text{det}(-\partial^2 + \beta + \ldots)^N.$
Minisuperspace wave functions

\[ y = \log \Psi[\beta] \] plotted as function of \( x \) for \( \beta(\omega_3) = x Y^m_{\ell}(\omega_3), \ell = 0, \ldots, 8 \), spherical harmonics on \( S^3 \). (Only) the \( \ell = 0 \) mode leads to a divergent wave function. 

[Anninos-FD-Harlow, Anninos-FD-Konstantinidis-Shaghoulian]

Wave function is function... of what?

Must use correct set of degrees of freedom, measure, etc before drawing conclusions about wave function divergences:

- $\psi(r) = \frac{e^{-r}}{r}$ diverges on $\mathbb{R}$ but converges on $\mathbb{R}^2$.
- Gauge invariance
- Other redundancies (holography!)

Most general free $U(N)$ invariant Lagrangian:

$$S = \int d^3x \left| \partial V^A \right|^2 + \int d^3x \, d^3y \, B(x, y) \, V^A(x) \bar{V}^A(y).$$

$B(x, y)$ bilocal collective field [Jevicki, Das-Jevicki]; can be expanded in primary fields coupling to sources.

Note mismatch d.o.f. “sources” $B(x, y)$ and “fields” $V^A(x)$. 
**Toy model illustration**

Toy model: Complex bosonic rectangular matrices $V_x^A$, $A = 1, \ldots, N$, $x = 1, \ldots, K$, coupling to hermitian matrix source $B_{xy}$:

$$
\Psi(B) = \int dV \ e^{-\text{Tr}(V\bar{V}) + i \text{Tr}(VB\bar{V})} = \det(1 + iB)^{-N}
$$

$$
\int dB |\Psi(B)|^2 = \int dB \det(1 + B^2)^{-N}.
$$

Under $B \to \lambda B$ with $\lambda \to \infty$:

$$
dB \propto \lambda^{K^2}, \quad \det(1 + B^2)^{-N} \propto \lambda^{-2NK}.
$$

$\Rightarrow \Psi(B)$ normalizable iff $K < 2N$, i.e. source $B$ d.o.f. < field $V$ d.o.f.

Alternatively: Keep in integral form, integrate out $B$:

$$
\int dB |\Psi(B)|^2 \propto \int dV \ dV' \prod_{ij} \delta((V\bar{V})_{ij} - (V'\bar{V}')_{ij})
$$

If $K > 2N$: product of redundant $\delta$-functions.
Degrees of freedom

**Extrapolation** to original $U(N)$ model with UV cutoff $\sim K$ spatial cells.

$\Rightarrow 2NK$ vector $V$ d.o.f., $K^2$ source d.o.f.

Continuum limit: $K \to \infty \Rightarrow$ sources vastly redundant.

Even after gauge fixing: 2 d.o.f. per spatial cell for each spin $s \geq 1$, infinite tower of spins.

In addition: $\partial^2 V = 0$, $V$ fermionic $\Rightarrow$ further reduction $V$ d.o.f.

**Conclusion:** Asymmetric parametrization fields/sources inadequate.

**Question:** Can we do better? Parametrize source as vector bilinears too, part of single dual QFT? Maybe, but recall commutator problem...

**Idea:** Berezin coherent states $\leadsto$ convergent integrals over sources.
Berezin coherent states

[Berezin, Das^2-Jevicki-Ye]: Quantum mechanics of $U(N)$ vector models = Kähler quantization of dual “source” phase space, generalizing spin $N$ Bloch sphere.

Standard radial quantization $U(N)$ fermionic vector model via mode expansion:

$$\{ a^A_p, (a^B_q)^\dagger \} = \delta^A_p \delta^B_q , \quad \{ b^A_p, (b^B_q)^\dagger \} = \delta^A_p \delta^B_q ,$$

where $A, B = 1, \ldots, N$ and $p, q = 1, \ldots, S$ (∼ angular momentum modes $\sim S$ ∼ number of points on codim-1 sphere)

Berezin coherent (or squeezed) states:

$$|\bar{Z}\rangle \equiv e^{\frac{1}{\sqrt{N}} Z_{pq} b^\dagger_p a^\dagger_q |0\rangle}, \quad (Z|\bar{W}) = \det(1 + \frac{1}{N} Z W^\dagger)^N .$$

So for normalized states $|Z\rangle \equiv \frac{1}{\sqrt{(Z|\bar{Z})}} |Z\rangle$, and for $N \to \infty$:

$$|\langle Z|\bar{W}\rangle|^2 \approx e^{-\Tr |Z - W|^2} .$$
Complex sources $Z_{pq}$ parametrize compact Kähler phase space with Kähler potential $\mathcal{K} = \log \det(1 + \frac{1}{N} ZZ^\dagger)$,

$$ds^2 = \text{Tr}[(1 + \frac{1}{N} ZZ^\dagger)^{-1} dZ]^2.$$

Decomposition of unity: We have $\int dZ \sqrt{G} \langle \bar{Z} \rangle \langle Z \rangle \propto 1$, i.e.

$$\int dZ \det(1 + \frac{1}{N} Z^\dagger Z)^{-2S} \langle \bar{Z} \rangle \langle Z \rangle \propto 1.$$

$N \to \infty$ fixed $Z, S \sim$ standard oscillator coherent states.

D.o.f. sources match vectors:

- compact phase space (diag. $Z$: (2-sphere)$^S$)
- data on codim-1 sphere position space $\leadsto$ holographic
**CFT state space**

Noted before: ghost $U(N)$ model inner product *not* $> 0$.

2-point function $\langle O(x)O(y) \rangle = \frac{c_O}{|x-y|^{2\Delta}}, \ c_O < 0$. Radial quantization: $O(x)\dagger = x^{-2\Delta} O(\frac{x}{x^2})^\ast$. Norm primary state $\langle O(0)\dagger|O(0)\rangle = c_O < 0$.

Mode expansion $V^A$ gives creation/annihilation operator algebra

$$\{a_p^A, (a_q^B)\dagger\} = \delta_p^A \delta_q^B, \quad \{b_p^A, (b_q^B)\dagger\} = -\delta_p^A \delta_q^B,$$

with $p = (\ell, m)$ angular momentum quantum numbers. (sign)

$\Rightarrow$ CFT “state” space $\mathbb{Z}_2$ graded with positive/negative norm for even/odd $b$-number.

**Bulk interpretation?** Evidently *not* global particle states.

[Anninos,Ng-Strominger,Jafferis-Lupsasca-Lysov-Ng]:

CFT states $\leftrightarrow$ bulk quasinormal modes.

[Note: Berezin construction modified: $(Z|\bar{Z}) = \det(1 - \frac{1}{N} ZZ\dagger)^N.$]
**Bulk reconstruction from CFT**

**Question:** CFT to bulk operator map $\sim$ [Hamilton-Kabat-Lifschytz-Lowe]? (dS: [Sharkar-Xiao])

**Approach:** AdS group theoretic construction of [Verlinde,Miyaji-Numasawa-Shiba-Takayanagi-Watanabe, Nakayama-Ooguri]

**Results:** E.g. planar: Most general formula consistent with symmetries:

$$\phi(\eta, x) \sim \sum_{\pm} \eta^{d-\Delta} \int dx' ((x - x')^2 - \eta^2 \mp i\epsilon)^{\Delta-d} O_\pm(x'),$$

$O_\pm$ primaries of dim $\Delta$. Has both $\eta^\Delta$, $\eta^{d-\Delta}$ falloffs.

Bulk QM $\Leftrightarrow [O_+(x), O_-(y)] \sim (x - y)^{-2\Delta} \rightsquigarrow$ commutator problem.

* Lor. AdS-CFT: inherited from pos./neg. freq. modes in CFT
  $$O_\pm(k) \sim \Theta(-k^2)\Theta(\pm k^0)O(k)$$

* dS-CFT? No time in boundary $\rightsquigarrow \times$ CFT knows nothing of bulk QM?
**Bulk reconstruction from CFT**

Not so fast: In static patch dS i.e.

\[
    ds^2 = -(1 - r^2)dt^2 + \frac{dr^2}{1 - r^2} + r^2 d\Omega^2
\]

CFT operator mode expansion in radial/cylinder quantization

\[
    \hat{O}(\tau, \Omega) = \sum_{\ell m} \sum_{\kappa} \hat{O}_{\ell m}^{\kappa} e^{\kappa \tau} \ Y_{\ell m}(\Omega),
\]

does map (with suitable variant of integration prescription) to:

\[
    \hat{\phi}(t, r, \Omega) = \sum_{\ell m} \sum_{\kappa_n = \pm (\Delta + \ell + 2n)} \hat{O}_{\ell m}^{\kappa_n} e^{-\kappa_n t} \ \psi_{\ell m}^n(r) \ Y_{\ell m}(\Omega)
\]

= static patch quasinormal mode expansion!

Moreover with \((\hat{O}_{\ell m}^{\kappa})^\dagger = (-)^m \hat{O}_{\ell, -m}^{\kappa}\):

\[
    [\hat{O}_{\ell m}^{\kappa}, (\hat{O}_{\ell m}^{\kappa})^\dagger] \sim \pm \delta_{\ell \ell'} \delta_{mm'} \delta_{\kappa \kappa'} + O\left(\frac{1}{N}\right)
\]

\[N \rightarrow \infty \sim QM \sim QNM\] quantization of [Strominger et al].
Conclusions

Way ahead: putting these pieces of the puzzle together

But I’m out of time.

Thank you!