

Topics in Analysis of Many Particle Systems

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This summer course will present some mathematical tools and concepts for the rigorous derivation and study of nonlinear partial differential equations (PDE's) arising from many-particle limits: (McKean-)Vlasov type equations, the vorticity formulation of the 2D incompressible Euler/Navier-Stokes equations, Boltzmann collision equations, nonlinear diffusion equations, quantum Hartree equations... Depending on time and interest it will include part or all of the following topics: the Liouville/Master equations of N -particle systems, the notion of empirical measures, the BBGKY hierarchy, the Hewitt-Savage theorem, the Dobrushin's stability estimate, the coupling method, the concepts of chaos and entropic chaos, the recent progresses on the mean-field limit, in particular, the relative entropy/modulated potential energy/modulated free energy methods as introduced in [6, 7, 11, 8].

Pre-requisites: Basics in measure theory, real and functional analysis, partial differential equations and probability.

Textbook: We will not use any textbook. But most materials will be based on the notes [4, 5] and recent articles [6, 7, 11, 8].

Time: TBA.

Hao Wu from Tsinghua will host a summer course by Yilin Wang from MIT as a part of THU-PKU-BNU Probability Webinar. Dates are August 3, 5, 10, 12. It would be better for me to start after her series. But it depends on your schedule.

Location: Online.

Examination: TBA. (Presentation/Oral exam/Final project?)

References

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- [5] P.-E. Jabin, A review of the mean field limits for Vlasov equations. Kinetic and Related models 7 (2014) 661–711.
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