



Trends in Modern Geometry 2019



2019.6.24 – 6.28



Tsinghua University, Beijing



The main purpose of Trends in Modern Geometry is to intensively discuss recent remarkable progress in the field of geometry, especially complex/Kaehler geometry, and Riemannian geometry. Another goal of this symposium is to give guidance for beginning graduate students, and especially to motivate them to further study these areas of geometry.



List of Speakers

- Vestislav Apostolov (UQAM, Montreal)
- Gao Chen (Princeton IAS)
- Jingyi Chen (British Columbia)
- Jingrui Cheng (Stony Brook)
- Tamas Darvas (Maryland)
- Tsz-Ho Fong (UST Hong Kong)
- Jianquan Ge (Beijin Normal U)
- Qi'an Guan (Peking)
- Jiyuan Han (Purdue)
- Yoshinori Hashimoto (Tokyo Tech)
- Julien Keller (Aix Marseille)
- Chi Li (Purdue)
- Martin Man-Chun Li (CU Hong Kong)
- Carlo Mantegazza (Napoli)
- Yuji Sano (Fukuoka)
- Zakarias Sjöström Dyrefelt (ICTP, Trieste)
- Jiaping Wang (Minnesota)
- Xiaowei Wang (Rutgers)
- Burkhard Wilking (Muenster)
- Hikaru Yamamoto (Tokyo U of Science)
- Shing-Tung Yau (Harvard)
- Ruobing Zhang (Stony Brook)



Organizers:

- Huai-Dong Cao, Lehigh University
- Shiu-Yuen Cheng, Tsinghua University
- Akito Futaki, Tsinghua University
- Jian Song, Rutgers University
- Jeff Viaclovsky, UC Irvine



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丘成桐数学科学中心
YAU MATHEMATICAL SCIENCES CENTER

SCHEDULE FOR TIMG 2019

- The symposium will be held at Yau Mathematical Sciences Center, JinChunYuan West Building, 3rd floor, Lecture Hall.

	Mon (June 24)	Tue (June 25)	Wed (June 26)	Thu (June 27)	Fri (June 28)
09:00-10:00	Jianquan Ge	Xiaowei Wang	Carlo Mantegazza	Vestislav Apostolov	Burkhard Wilking
10:00-10:30	Group Photo+Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10:30-11:30	Tsz-Ho Fong	Chi Li	Tamas Darvas	Yuji Sano	Jiyuan Han
11:30-13:30	Lunch	Lunch	Lunch	Lunch	Lunch
13:30-14:30	Man-Chun Li	Hikaru Yamamoto		Julien Keller	Ruobing Zhang
14:30-14:50	Break	Break		Break	Break
14:50-15:50	Yoshinori Hashimoto	Jingrui Cheng		Jingyi Chen	Gao Chen
15:50-16:10	Break	Break		Break	Break
16:10-17:10	Qi'an Guan	Sjostrom Dyrefelt		Jiaping Wang	Shing-Tung Yau

- Free Discussion for afternoon on Wednesday (June 26)
- The Chairs

	Mon (June 24)	Tue (June 25)	Wed (June 26)	Thu (June 27)	Fri (June 28)
Morning	Akito Futaki	Xiaohua Zhu	Huai-Dong Cao	Shu-Cheng Chang	Jeff Viaclovsky
Afternoon	Toshiki Mabuchi	Jian Song		Haizhong Li	Zizhou Tang

TITLES AND ABSTRACTS

Monday (June 24)

JIANQUAN GE

(BEIJING NORMAL UNIVERSITY)

Isoparametric polynomials and sums of squares

Hilbert's 17th problem asks that whether every nonnegative polynomial can be a sum of squares of rational functions. It has been answered affirmatively by Artin. However, as to the question whether a given nonnegative polynomial is a sum of squares of polynomials is still a central question in real algebraic geometry. In this talk, we solve this question completely for the nonnegative polynomials (associated with isoparametric polynomials, initiated by E. Cartan) which define the focal submanifolds of the corresponding isoparametric hypersurfaces.

FREDERICK TSZ-HO FONG

(HONG KONG UNIVERSITY OF SCIENCE AND
TECHNOLOGY)

Curvature Estimates of Long-Time Solutions to the Kähler-Ricci Flow

The speaker will discuss the local curvature estimates of the Kähler-Ricci flow on compact Kähler manifolds with semi-ample canonical line bundles. On such a manifold, the Kähler-Ricci flow has long-time solutions and its convergence and singular behaviors have been widely studied by various authors. In this talk, the speaker will discuss his works on this topic, in particular showing that the set of fibers (either singular or regular) on which the Riemann curvature blow up along the flow is an invariant set independent of the choice of initial

Kähler metric. The talk is based on two joint works, one with Zhou Zhang, another with Yashan Zhang. The research conducted is partially supported by Hong Kong RGC Grants #26301316 and #16300018.

MARTIN MAN-CHUN LI

(CHINESE UNIVERSITY OF HONG KONG)

On Min-Max Constructions of Free Boundary Minimal Surfaces

In this talk, I will discuss some recent progress on various min-max constructions of free boundary minimal hypersurfaces in Riemannian manifolds with boundary.

YOSHINORI HASHIMOTO

(TOKYO INSTITUTE OF TECHNOLOGY)

Variational Aspects of the Kobayashi-Hitchin Correspondence and the Quot-Scheme Limit

The Kobayashi-Hitchin correspondence, proved by Donaldson and Uhlenbeck-Yau by using the nonlinear PDE theory, states that the existence of Hermitian-Einstein metrics on a holomorphic vector bundle is equivalent to an algebro-geometric stability condition. We present some results that exhibit an explicit link between differential and algebraic geometry in the above correspondence, from a variational point of view. The key to such results is an object called the Quot-scheme limit of Fubini-Study metrics, which is used to evaluate certain algebraic 1-parameter subgroups of Hermitian metrics by using the theory of Quot-schemes in algebraic geometry. Joint work with Julien Keller.

QI'AN GUAN

(PEKING UNIVERSITY)

Multiplier Ideal Sheaves and L^2 Extension Problem

In this talk, we will recall some recent progress related to multiplier ideal sheaves and L^2 extension theorem with optimal estimate. This is based on joint work with Professor Xiangyu Zhou.

Tuesday (June 25)

XIAOWEI WANG

(RUTGERS UNIVERSITY - NEWARK)

Existence of Algebraic Moduli Space for a Global Quotient

In this talk, we will discuss the condition on the existence of algebraic structure on quotients of a proper topological space by compact Lie groups. This can be regarded as a generalization of the classical Guillemin-Sternberg-Kirwan theory.

CHI LI

(PURDUE UNIVERSITY)

The Uniform Version of Yau-Tian-Donaldson Conjecture for Singular Fano Varieties

I will explain a proof of the following version of Yau-Tian-Donaldson conjecture: A singular \mathbb{Q} -Fano variety with a discrete automorphism group admits a Kähler-Einstein metric if and only if it is uniformly K-stable. This result doesn't require extra constraint on singularities. We achieve this by modifying Berman-Boucksom-Jonsson's variational approach (in the smooth case) with

appropriate perturbative and non-Archimedean estimates. The idea of using the perturbation is motivated by our previous work, in which we proved the full Yau-Tian-Donaldson conjecture for any \mathbb{Q} -factorial Fano varieties admitting crepant resolutions. This is joint work with Gang Tian and Feng Wang.

HIKARU YAMAMOTO

(TOKYO UNIVERSITY OF SCIENCE)

An Epsilon Regularity Theorem for Line Bundle Mean Curvature Flow

The line bundle mean curvature flow was defined by A. Jacob and S.-T. Yau to obtain deformed Hermitian Yang-Mills metrics on a line bundle over a Kähler manifold. In this talk, I would like to explain an epsilon regularity theorem for the line bundle mean curvature flow and its philosophy. This is joint work with X. Han. To explain the outline of the proof, I would like to introduce a scale-invariant monotone quantity, a notion of self-shrinkers and the Liouville type theorem for those self-shrinkers.

JINGRUI CHENG

(STONY BROOK UNIVERSITY)

Existence of Constant Scalar Curvature Kähler Metrics and Properness of Mabuchi Energy

I will explain the a priori estimates for the equation of constant scalar curvature Kähler metrics on a compact Kähler manifold, and show that an entropy bound will imply higher derivative bound. As an application, we show that the properness of Mabuchi energy implies the existence of constant scalar curvature Kähler metrics. This is joint work with Xiuxiong Chen.

ZAKARIAS SJÖSTRÖM DYREFELT

(THE ABDUS SALAM INTERNATIONAL CENTRE
FOR THEORETICAL PHYSICS (ICTP), TRIESTE)

Lower Bounds of the Twisted K-Energy Functional

In this talk we give an explicit formula for the optimal lower bound of the pluripolar part of the twisted K-energy functional, in the sense of the optimal constant for which it is proper. We further discuss applications to Calabi dream manifolds, i.e. compact Kähler manifolds that admit constant scalar curvature Kähler metrics in every Kähler class.

Wednesday (June 26)

CARLO MANTEGAZZA

(UNIVERSITÀ DEGLI STUDI DI NAPOLI FEDERICO II)

Evolution by Curvature of Networks in the Plane

I will present the state-of-the-art of the problem of the motion by curvature of a network of curves in the plane, discussing existence, uniqueness, singularity formation and asymptotic behavior of the flow.

TAMAS DARVAS

(UNIVERSITY OF MARYLAND)

Geodesic Stability, the Space of Rays and Uniform Convexity in L^p Mabuchi Geometry

We show that the L^p Mabuchi metric spaces are uniformly convex for $p > 1$, implying that these spaces are uniquely geodesic. Using this result, we describe the metric geometry of L^p Mabuchi geodesic rays associated to a Kähler manifold. We point out that any L^p geodesic ray can be approximated by rays of potentials with bounded Laplacian, while recovering the

slope of the K-energy in the limit. Finally, we use these results to verify the optimal version of Donaldson's geodesic stability conjecture on existence of constant scalar curvature Kähler metrics. This is joint work with C.H. Lu.

Thursday (June 27)

VESTISLAV APOSTOLOV

(UNIVERSITÉ DU QUÉBEC À MONTRÉAL)

The CR Geometry of Weighted Extremal Kähler and Sasaki Metrics

I will discuss an equivalence between Einstein-Maxwell 4-manifolds obtained from Kähler metrics, and extremal Kähler 4-manifolds in the sense of Calabi. The corresponding pairs of Kähler metrics arise as transversal Kähler structures of Sasaki metrics on the same CR 5-manifold. The correspondence extends to higher dimensions, using a notion of weighted extremal Kähler metric introduced by Lahdili, thus providing a link with the notion of K-stability associated to Kähler cone metrics corresponding to a Sasaki polarization, introduced by Collins-Székelyhidi. This talk is based on a joint work with David Calderbank.

YUJI SANO

(FUKUOKA UNIVERSITY)

A Moment Map Model for Relative Balanced Metrics

When a moment map does not have any zero, a natural approach studied by Székelyhidi is to find the critical points of the squared norm of the moment map. In this talk, I consider another approach and apply it to the study of the relative balanced metrics and the quantization of the extremal metrics. This talk is essentially based on the joint work with Carl Tipler.

JULIEN KELLER

(AIX-MARSEILLE UNIVERSITY)

Constant Scalar Curvature Kähler Metrics with Conic Singularities

We will review some recent progress on the study of constant scalar curvature Kähler metrics with conic singularities along a divisor. In particular, we will discuss the existence of such special metrics, focusing on the particular case of projective bundles where constructions can be done. Moreover, when considering projective manifolds, we will discuss the relationship with the algebraic notion of log K-stability. (Joint work with Kai Zheng)

JINGYI CHEN

(UNIVERSITY OF BRITISH COLUMBIA)

Compactness of the Space of Hamiltonian Stationary Lagrangian Submanifolds

We will discuss convergence of a sequence of closed Hamiltonian stationary Lagrangian submanifolds in C^n with uniform bounds on volume and extrinsic total curvature. This is joint work with M. Warren.

JIAPING WANG

(UNIVERSITY OF MINNESOTA)

Green's Function Estimates and Applications

The talk aims to discuss some recent joint work with Munteanu and Sung concerning sharp integral estimate on the minimal positive Green's function. Applications to steady Ricci solitons and holomorphic maps will also be mentioned.

Friday (June 28)

BURKHARD WILKING

(UNIVERSITÄT MÜNSTER)

The Topology of Fixed Point Components of Tori's Actions
in Positively Curved Manifolds

We show that each fixed point component of an isometric torus action of a 5 torus has the rational cohomology of a rank one symmetric space. We give various applications.

JIYUAN HAN

(PURDUE UNIVERSITY)

Existence and Compactness Theory for ALE Scalar-Flat
Kähler Surfaces (Joint with J. Viaclovsky)

Our main result in this article is a compactness result which states that a noncollapsed sequence of asymptotically locally Euclidean (ALE) scalar-flat Kähler metrics on a minimal Kähler surface whose Kähler classes stay in a compact subset of the interior of the Kähler cone must have a convergent subsequence. As an application, we prove the existence of global moduli spaces of scalar-flat Kähler ALE metrics for several infinite families of Kähler ALE spaces.

RUOBING ZHANG

(STONY BROOK UNIVERSITY)

Metric Geometry and Analysis of Collapsed Einstein Spaces

This talk centers around recent progress on the geometry of collapsed Einstein manifolds. In a collapsing sequence, we are concerned with the limiting behaviors of metric tensors, measures and topology and how these elements capture the regular/singular geometry of the collapsing sequence. The first part is to introduce some new regularity theorems for collapsed Einstein manifolds in higher dimensions and their applications in studying the Einstein limits. Next, we will exhibit some entirely new constructions of collapsed Ricci flat spaces which limit to a closed interval. This amounts to the Type II degeneration in complex algebraic geometry. From the metric geometric point of view, a crucial new ingredient is the singular iterated collapsing structure on Ricci-flat manifolds. In particular, such collapsing scenario in real dimension 4 gives singular nilpotent Killing structures on K3 surfaces.

GAO CHEN

(INSTITUTE FOR ADVANCED STUDY)

On J-equation

In this talk, I will explain my recent result that the solvability of the J-equation is equivalent to uniform J-stability. I will also explain its application to the constant scalar curvature Kähler problem and the deformed Hermitian-Yang-Mills equation.

SHING-TUNG YAU

(HARVARD UNIVERSITY)

Stability and nonlinear PDEs in mirror symmetry

Mirror symmetry, the special Lagrangian and deformed Hermitian-Yang-Mills equations, variational approach to dHYM, connections with algebraic geometry, and stability conditions, applications to symplectic geometry, and future directions.