# Superluminal Neutrinos and Special Relativity with de Sitter Space-time Symmetry

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The OPERA collaboration recently reported evidence of superluminal behavior for muon neutrinos $\nu_\mu$ with energies of a few tens of GeVs [1]. The arrival time of the $\nu_\mu$ neutrino with average energy of 17 GeV is earlier by $\delta t = (60.7 \pm 6.9_{stat} \pm 7.4_{sys})$ ns. This translates into a superluminal propagation velocity for neutrinos by a relative amount

$$\delta c_\nu = (v_\nu - c)/c = (2.48 \pm 0.28_{stat} \pm 0.30_{sys}) \times 10^{-5} \tag{1}$$

with significance level of $6\sigma$. This datum is consistent with the earlier MINOS experiment [2] and FERMILAB79 experiment [3].

This would be the most significant discovery in fundamental physics over the last several decades because OPERA datum definitely indicates $v_\nu > c$. It directly challenges the Einstein's Special Relativity (E-SR). As is well known that E-SR has been one cornerstone of modern physics, well-established by innumerable experiments and observations. An outstanding feature of E-SR is a universal upper limit of speed, namely the light speed $c$ in vacuum. However, astonishingly, this speed record is broken by OPERA experiment. Furthermore, in the E-SR frame work, Cohen and Glashow [4] and Bi, *et al* [5] argued that such superluminal neutrinos should lose energy by producing $e^+e^-$ pairs, through $Z^0$ mediated processes analogous to Cherenkov radiation. But soon, the ICARUS collaboration reported that there is no such sort of energy loss signals that were observed [6]. In this circumstance, we feel that it is time to re-examine the principle of the Special Relativity more healthily and more carefully. Motivated by both the results of OPERA experiment and the results of ICARUS's, in this paper, we attempt to solve the puzzle arisen from these recent experiments by means of the Special Relativity with de Sitter space-time symmetry (dS-SR) [7–9].

Fundamentally, the Special Relativity (SR) is a theory with regard to the global space-time symmetry. Such symmetry is the foundation and the starting point for upbuilding whole physics. As is well known that the space-time metric in E-SR is $\eta_{\mu\nu} = diag\{+,-,-,-\}$. The most general transformation to preserve metric $\eta_{\mu\nu}$ is global Poincaré group (or inhomogeneous Lorentz group $ISO(1,3)$). It is well known also that the Poincaré group is the limit of the de Sitter group with sphere radius $R \to \infty$. Thus a natural question arisen from this fact is whether there exists or not another type of de Sitter transformation with $R \to finite$ which also leads to a special relativity theory. In 1970's, K.H. Look (Qi-Keng Lu) and his collaborators Z.L. Zou and H.Y. Guo have pursued this question and got highly nontrivial positive answer, and then formulated the mathematic structure of the Special Relativity with global de Sitter space-time symmetry [7, 8]. To the best of our knowledge, Ref. [8] is the first publication to explore SR theory by means of the global de Sitter space-time symmetry, i.e., dS-SR. In 2005, Yan, Xiao, Huang, Li [9] performed Lagrangian-Hamiltonian formulism for dS-SR with two universal constants $c$ and $R$ , and suggested the quantum mechanics of dS-SR. Ref. [9] is the base of our investigation in this present paper.

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A meaningful and deep physical question is that what is the space-time symmetry for the real world? People could doubt that E-SR corresponding to dS-SR with $R \to \infty$ may be an approximation of dS-SR with large enough $R$. To get the answer, one should pursue the physical effects beyond E-SR. The anomaly of OPERA belongs to such sort of physics. Since the puzzle arisen from the experiments will be solved by means of dS-SR in this paper, the OPERA anomaly should be an experiment to determine the space-time symmetry of the real physical world. So it is significant.

In recent work of [10], we accept Einstein's hypotheses that the photon could be treated as a massless particle in the Special Relativity mechanics, and its velocity $c_{photon}$ is the physical energy-momentum propagating speed of light in vacuum. We do not assume $c_{photon} = c$ beforehand. The wave phase propagating velocity of light in vacuum is $c_{wave} = \lambda\nu$. The relationships between $c$, $c_{photon}$ and $c_{wave}$ in both E-SR and dS-SR are carefully studied in the paper. In SR, universal parameter $c$ is required to be independent of the reference systems. And in other hand the famous null experiments of Michelson-Morley shown that the light wave velocity $c_{wave} = \lambda\nu$ is independent of the reference systems with very high accuracy. Thus, Einstein's outstanding assumption of $c = c_{wave}$ is sound and of the foundation of both E-SR and dS-SR. What is new in this paper is that the $c_{photon}$ is derived from the Noether chargers generated from the SR's space-time symmetries. We will reveal in the paper that $c_{photon} = c = c_{wave}$ for E-SR, and, however, $c_{photon} > c = c_{wave}$ for SO(4,1) dS-SR. This is an interesting result because it comes from the SR space-time symmetry principle, and there are no *ad hoc* considerations that are involved. Since $m_\nu$ is rather small, it is easy to achieve conclusion of $c_{photon} > v_\nu > c$ when $E_\nu$ is large enough. Furthermore, the kinematic calculations based on dS-SR dispersion relation show that the Cherenkov-like process of $\nu_\mu \to \nu_\mu + e^+ + e^-$ is forbidden. Consequently, under SR principle the OPERA anomaly on superluminal neutrinos well be interpreted by means of dS-SR naturally in this present paper.

In Special Relativity with de Sitter spacetime symmetry we have the Beltrami metric $ds^2 = \Sigma_{\mu\nu} B_{\mu\nu} dx^\mu \otimes dx^\nu$ of the global spacetime coordinates $x^\mu, \mu = 0, 1, 2, 3$:

$$B_{\mu\nu} = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{\eta_{\mu\lambda}\eta_{\nu\rho}x^\lambda x^\rho}{R^2\sigma(x)^2}, \quad with \quad \sigma(x) := 1 - \frac{1}{R^2}\eta_{\mu\nu}x^\mu x^\nu, \tag{2}$$

where the speed of light $c$ and the radius $R$ of the pseudo-sphere in the de Sitter space are universal constants.

The Lagrangian of a free particle in dS-SR reads

$$L_{dS}(t, x^i, \dot{x}^i) = -m_0 c\sqrt{B_{\mu\nu}(x)\dot{x}^\mu \dot{x}^\nu} \tag{3}$$

The equation of motion is

$$v^i = \dot{x}^i = const \tag{4}$$

Thus the coordinates for de Sitter spacetime can be used as an inertial frame metric.

In choosing spacetime coordinates we use the Big Bang (BB) as the natural origin. In the Earth laboratory we are at the time $t_0 = 13.7$Gy and $x_0 = x(t_0) = 0$. The three dimensional space is always isotropic and homogeneous by the Copernicus principle.

In the de Sitter spacetime we have ten parameters transformations preserving the Beltrami metric:

$$\begin{aligned} x^\mu \xrightarrow{dS} \tilde{x}^\mu &= \pm\sigma(a)^{1/2}\sigma(a,x)^{-1}(x^\nu - a^\nu)D^\mu_\nu, \\ D^\mu_\nu &= L^\mu_\nu + R^{-2}\eta_{\nu\rho}a^\rho a^\lambda(\sigma(a)+\sigma^{1/2}(a))^{-1}L^\mu_\lambda, \quad L := (L^\mu_\nu) \in SO(1,3), \\ \sigma(x) &= 1 - \frac{1}{R^2}\eta_{\mu\nu}x^\mu x^\nu, \ \sigma(a,x) = 1 - \frac{1}{R^2}\eta_{\mu\nu}a^\mu x^\nu. \end{aligned} \tag{5}$$

It gives 10 conserved charges:

$$p^i_{dS} = m_0\Gamma\dot{x}^i, \quad E_{dS} = m_0c^2\Gamma, \qquad K^i_{dS} = m_0c\Gamma x^i - tp^i_{dS}, \ L^i_{dS} = -\epsilon^i{}_{jk}x^j p^k_{dS}.$$

Here $E_{dS}, \mathbf{p}_{dS}, \mathbf{L}_{dS}, \mathbf{K}_{dS}$ are conserved physical energy, momentum, angular-momentum and boost charges respectively, and $\Gamma^{-1}$ is $\sigma(x)\frac{ds}{cdt}$.

It is straightforward to check the identity of $\sigma^2(x)B_{\mu\nu}(x)p^\mu_{dS}p^\nu_{dS} = m_0^2c^2$. Then we have the dispersion relation for dS-SR [9]

$$E^2_{dS} = m_0^2c^4 + \mathbf{p}^2_{dS}c^2 + \frac{c^2}{R^2}(\mathbf{L}^2_{dS} - \mathbf{K}^2_{dS}). \tag{6}$$

The OPERA experiment can be understood in the framework of de Sitter Special Relativity. We consider neutrinos as free massive particles. Its speed can be derived through the dispersion relation. Suppose the OPERA neutrinos moving trajectory is $\{x^1 = x(t), x^2 = 0, x^3 = 0\}$, then

$$v_{dS} \equiv \dot{x}(t) = \frac{c^2p_{dS}}{E_{dS}}, \quad with \ \ E_{dS} = \frac{m_0c^2}{\sqrt{1 - (\frac{v_{dS}}{c})^2 + (\frac{x_0 - v_{dS}t_0}{R})^2}}, \tag{7}$$

where $t_0$ and $x_0$ are the OPERA neutrino moving's initial time and space location, i.e., $t_0 \simeq 13.7Gy, \ \ x_0 = x(t_0) \simeq 0$.

We have the formulae of neutrino velocity:

$$v_{dS} = c\sqrt{\frac{1 - \frac{m_0^2c^4}{E^2}}{1 - \frac{c^2t_0^2}{R^2}}} \tag{8}$$

We see when $E$ is large enough we could have $v_{dS} > c$.

In the OPERA experiment we have $E = 13.9GeV$, $m_0 = 2eV$ and

$$\delta v_\nu = \frac{v_\nu - c}{c} = (2.48 \pm 0.28_{stat} \pm 0.30_{sys}) \times 10^{-5}. \tag{9}$$

We have: $\frac{v_{dS}}{c} = 1 + \delta v_\nu = 1 + \frac{1}{2}\frac{c^2t_0^2}{R^2} - \frac{1}{2}\frac{m_0^2c^4}{E^2}$. By neglecting the higher order term we have:

$$\delta v_{dS} = \frac{1}{2}\frac{c^2t_0^2}{R^2}.$$

The radius $R$ of the pseudo-sphere can be estimated as

$$R = \frac{ct_0}{\sqrt{2\delta v_{dS}}} = (1.95 \pm 0.11 \pm 0.12) \times 10^{12} l.y. \tag{10}$$

Thus we reach the conclusion that OPERA anomaly is in agreement with the prediction of dS-SR with $R \simeq 1.95 \times 10^{12} l.y.$

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