Slopes and Harder-Narasimhan Filtrations in Arithmetic and Geometry

Professor Jean-BenoîtBost
Département de Mathématiques
Université de Paris-Sud-Paris 11, Orsay, France

E-mail: jean-benoit.bost@math.u-psud.fr

Mathematical Sciences Center, Tsinghua University,Beijing, P. R. China
August, 2010

Contents

Preface

These notes are the faithful record of a course “Slopes and Harder-Narasimhan Filtrations in Arithmetic and Geometry”, which is given by Prof. Jean-BenoîtBost during July and August in 2010, in the Mathematical Sciences Center, Tsinghua University. All the manuscript is taken by Chunhui Liu (E-mail: lchws@163.com) from Department of Mathematical Sciences, Tsinghua University.

The theory of slopes and of Harder-Narasimhan filtrations——initially devised for the study of vector bundles on projective algebraic curves and then on projective varieties——makes sense in a general abstract framework, which turns out to have applications in very diverse contexts, including the theory of p-adic representations (Fontaine), analytic differential equations in the complex and p-adic domains, the theory of shtukas (Drinfeld-Lafforgue), Diophantine geometry and diophantine approximation on algebraic groups, Hermitian vector bundles in Arakelov geometry,…In these various domains, the general formalism of slopes and Harder-Narasimhan filtrations provides a simple and unified view of constructions and proofs, and turns out to be very helpful both conceptually and technically.

During the work of recording, Mathematical Sciences Center offered a lot of help. Prof. Bost explained some details after his course and discussed some related topics with us. At the same time, I am grateful toGuangyu Zhu from Tsinghua University, RunpuZong from Princeton University and Haoyu Hu from Nankai University for their help. Finally, I want to thank Cong Xue from Ecole Polytechniqueespecially. She checked the manuscript and corrected mistakes with me after every course during that summer vocation. As it is a “manuscript”, mistakes may be met when reading it, so all correction and suggestion for it are welcome.

Reference

[1] W. Fulton, algebraic curves: An Introduction to Algebraic Geometry, Addison Wesley Publishing Company (December 1974).

[2] J. Le Potier, Lectures on vector bundles, Cambridge studies in mathematics 54.

[3] J. Neukirch, Algebraic number theory, Springer-Verlag 1999.

[4] Robin Hartshorne, Algebraic Geometry (GTM 52), Spinger-Verlag 1977.

[5] Y. André, Slope Filtration, Arxiv: 0812.3921v2.

[6]G. Harder, M. Narasimhan, On the cohomology groups of moduli spaces of vector bundles on curves, Math. Ann. 212 (1975) 215-268.

[7] J. P. Serre, Géométriealgébriqueetgéométrieanalytique, Annales de l'institut Fourier, tome 6 (1956), p. 1-42.

[8]Jean-Benoit Bost, Klaus Kuennemann, Hermitian vector bundles and extension groups on arithmetic schemes. I. Geometry of numbers, Arxiv: math/0701343v1.

[9] Jean-Benoit Bost, Klaus Kuennemann, Hermitian vector bundles and extension groups on arithmetic schemes. II. The arithmetic Atiyah extension, Arxiv:0807.4374.